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Příspěvek k studiu ontogeneze nuculanidních mlžů

Contribution to the Study of the Ontogeny of Nuculanid Pelecypods

(S 5 tabulkami a 8 obrázky v textu)

(With 5 tables and 8 text figures)

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Při studiu nuculanidních svrchnokarbonských mlžů (*Pelecypoda*) hornoslezské předhlubně (K u m p e r a, O. — P r a n t l, F. — R ů ž i č k a, B., 1960; R ů ž i č k a, B. — B o j k o w s k i, J., 1960) setkávali jsme se podobně jako dřívější autoři s určitými potížemi, které spočívaly v tom, že jsme nemohli uvnitř námi studovaných druhů sestavit přirozené ontogenetické řady. Proto jsme učinili zajímavý pokus s holotypem druhu *Polidevcia hrebnickii* R ů ž i č k a — B o j k o w s k i, spočívající v tom, že jsme rozkreslili jednotlivá růstová stadia holotypu, respektive pravé misky jej představující, podle jednotlivých přírůstkových plátů. Získali jsme tak celkem 24 samostatných obrysových křivek, které odpovídají postupným růstovým stadiím studovaného jedince.

Tato růstová stadia lze seskupit do tří až čtyř ontogenetických fází, které předběžně označujeme jako fázi b r e f i c k o u, n e a n i c k o u, e f e b i c k o u a g e r o n t i c k o u.

Ontogeneticky nejmladší f á z e b r e f i c k á odpovídá stadiím, ve kterých ještě nedošlo k vytvoření charakteristického rostra, a bylo proto možno ji označit i jako f á z i p r e r o s t r á l n í. Tato fáze odpovídá na studovaném jedinci růstovému stadiu 1—6.

Další f á z e n e a n i c k á, odpovídající růstovému stadiu 7—8, je charakterizována projevením tendence k tvorbě rostra a bylo by ji možno označit také jako i n i c i á l n í f á z i r o s t r á l n í.

E f e b i c k á f á z e, která počíná růstovým stadiem 9, odpovídá ontogenetické fázi, ve které vytváření rostra se stalo již znakem dominantním. Proto je možno označit tuto fázi jako f á z i r o s t r á l n í.

Konečná f á z e g e r o n t i c k á je fází senilní a odpovídá snížení rychlosti celkového růstu misky.

Uvedené ontogenetické fáze růstu studované misky druhu *Polidevcia hrebnickii* pravděpodobně odpovídají souběžně probíhajícímu ontogenetickému vývoji měkkých tělesných částí daného jedince. Za předpokladu, že tyto ontogenetické fáze a jejich jednotlivá růstová stadia odpovídají rekapitulaci vývoje fylogenetického, můžeme usuzovat, že nuculanidní formy procházely ve svém vývoji fází, ve které ještě k vývoji rostra nedošlo, a že tedy pocházejí z morfologicky odchylně utvářených ancestrálních forem. Dále nepovažujeme za vyloučeno, že počátek tendence k vytváření rostra byl podmíněn změnami genetického charakteru, za spolupůsobení změněných ekologických faktorů.

Účelem této studie byla též snaha ukázat nezbytnost podrobné biometrické analýzy ontogenetického vývoje jednotlivých forem považovaných běžně za samostatné druhy. Doufáme, že zjištění rozdílného charakteru přibývání délky a výšky u nuculanidních mlžů mohlo by se stát objektivním kriteriem pro stanovení druhu.

Potvrzení tohoto názoru může ovšem přinést jen další zevrubné studium v tomto směru.

Současně považujeme za neudržitelné používání běžné určovací metody, které při druhovém srovnávání různých forem nepřihlíží k morfologickým diferencím podmíněným různými ontogenetickými stadii srovnávacího materiálu.

Contribution to the Study of the Ontogeny of Nuculanid Pelecypods

Abstract

The question of applicability or inapplicability of certain parameters for the biometrical analysis of nuculanid pelecypods is discussed in the present paper. Furthermore, the growth regularity of the characteristic proportions during the ontogeny of the studied member of the family (*Polidevcia hrebnickii* Růžička-Bojkowski, 1960) is followed.

In the course of the study of nuculanid pelecypods of the Upper Silesian fore-deep (Kumpera, O. — Prantl, F. — Růžička, B., 1960; Růžička, B. — Bojkowski, J., 1960) the authors, as well as preceding students, met with certain difficulties, which consisted in the impossibility of arranging natural ontogenetic series within the species under study. Therefore, an interesting experiment was executed. The individual growth degrees of the holotype of *Polidevcia hrebnickii* Růžička — Bojkowski (right valve) were drawn according to the individual growth lines. On the whole, twenty four outlines corresponding to the successive growth degrees of the studied specimen were obtained.

These growth degrees can be grouped into three or four ontogenetic stages which are called preliminarily the brephic, neanic, ephebic and gerontic stage.

The first, ontogenetically youngest brephic stage corresponds to the holometric degrees in which the formation of the characteristic rostrum did not yet take place. It may be called prerostral stage as well. This stage corresponds to the growth degrees 1—6 of the specimen under study.

The next, neanic stage, corresponding to the growth degrees 7—8, is characterized by the allometric tendency to form a rostrum and, therefore, it may be called initial rostral stage as well.

The ephebic stage, beginning by the growth degree 9, represents the ontogenetic stage in which the formation of the rostrum becomes already a dominant character. Consequently, it may be also called the rostral stage (see fig. 1).

The final, gerontic stage is a senil stage representing a reduction of the general growth rate of the valve.

The mentioned ontogenetic stages of the valve of *Polidevcia hrebnickii* need not correspond (but probably do) to the simultaneous ontogenetic development of the soft part of the body.

On the presumption that these ontogenetic stages and their individual growth degrees correspond to the recapitulation of the phylogenetic development. we can deduce that the nuculanid forms of pelecypods passed in the course of their development through a stage in which the formation of the rostrum did not yet take place and, consequently, that they originate in some morphologically different ancestral forms.

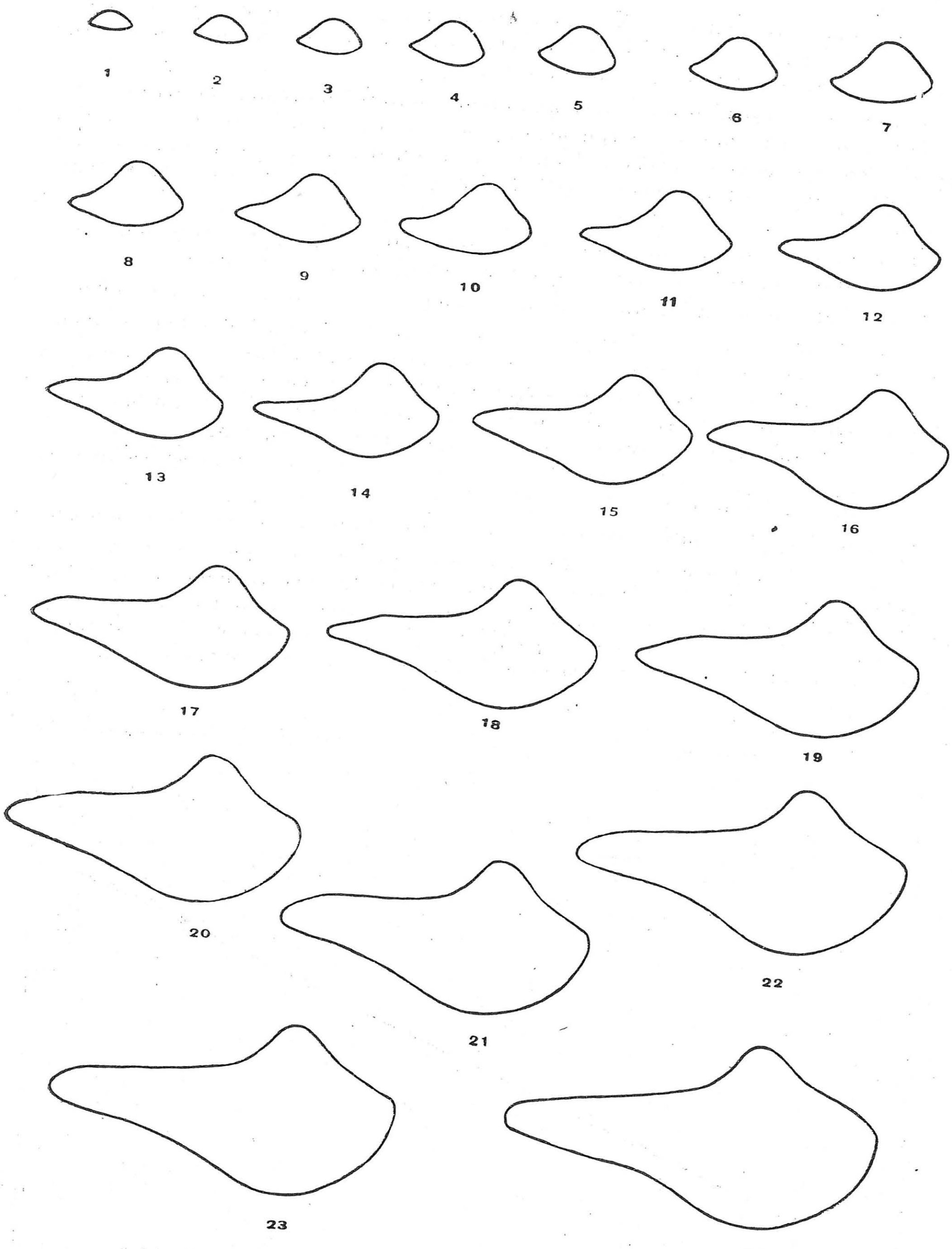

Fig: 1

Furthermore, we do not exclude the possibility that the beginning of the tendency to form a rostrum was caused by the changes of genetical character under the influence of altered ecological conditions.

On the specimen which was drawn and divided into the individual growth degrees according to the prominent morphological course of the outline, we tried to verify as to what extent the statistical biometrical methods, used several times by us, are able to express these changes (R ů ž i č k a, B. — P r a n t l, F., 1958, 1959, 1960).

Therefore, the anterior and posterior extreme point and the valve height were determined in all growth degrees, which were enlarged at a constant ratio.

First of all the v/l ratio was studied, in which l is the parameter $\overline{VP}$ (highest point of the beak—anterior extreme point) plus the parameter $\overline{VZ}$ (highest point of the beak—posterior extreme point); v is the height of the valve. The measured values were plotted into a graph (fig. 2), from which it is evident that they demonstrate a strong rectilinearity. Rectilinear regression formulae were computed for these points (tab. I) and plotted into the graph. From all that was stated above it can be presumed that the different growth degrees of the same species treated by the above described method are linearly related.

Furthermore, the relation between v and d was studied, d being the connecting line of $\overline{ZP}$ and v the height of the valve. This relation was also plotted into a graph (fig. 3). The result was a system of points, through which a single straight line could not be drawn. By the computed rectilinear regression formulae (tab. 2a, b, c) it was ascertained that the points 1—7 (the numbers also denote the individual growth degrees), 7—18 and 18—24 lie nearly precisely on straight lines of different

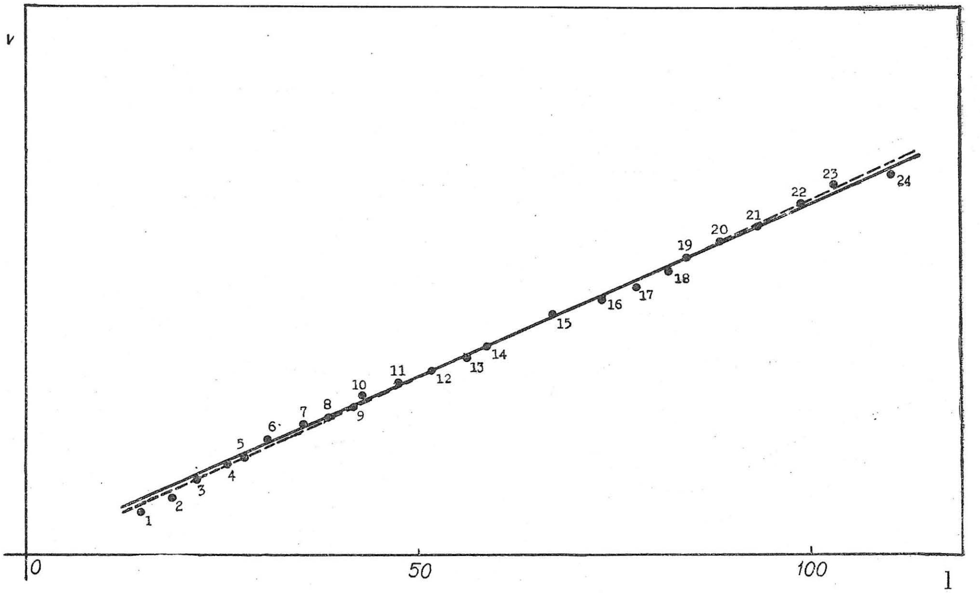

Fig. 2

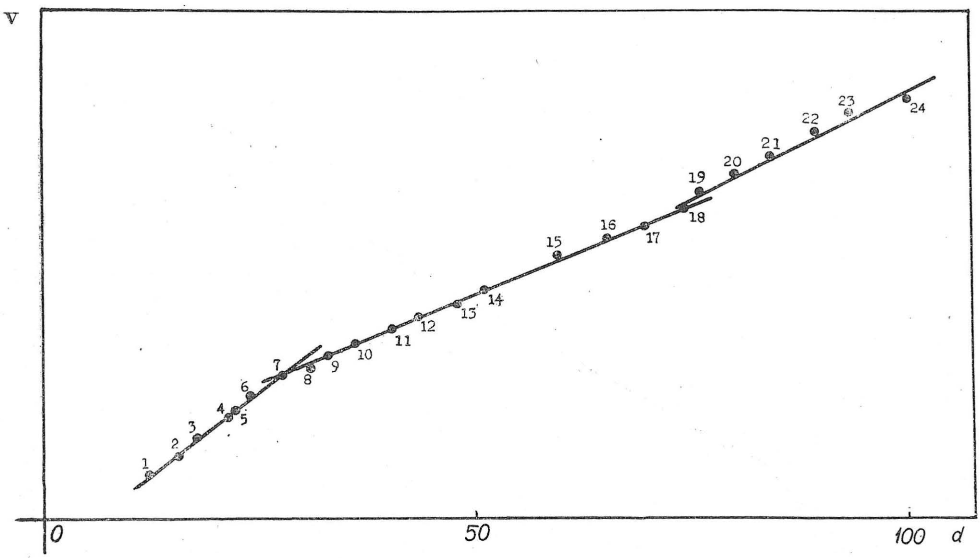

Fig. 3

slopes. The statistical comparison of these three series can be carried out on the assumption that each of them is taken for an individual statistical sample.

In comparing the differences between the regression coefficients of each two samples and the determinative deviation of the correlation coefficients, it becomes evident that the differences between the samples are statistically significant. (In the samples 1—7 and 7—18 $\frac{db}{\sigma_{db}} = 16{,}02$; in the samples 7—18 and 18—24 $\frac{db}{\sigma_{db}} = 5{,}126$, db being the difference between the regression coefficients of the two samples and σ_{db} the determinative deviation of the two correlation coefficients). From what was stated above the following conclusion can be drawn:

If an author used this method for a number of valves of different sizes, which represent different ontogenetic stages of the studied species (it should be noted that to date this is the usual procedure in the study of pelecypods), he would find mathematical proof that these forms are not conspecific, while the opposite is the thruth.

Likewise, the relation between v and z was studied, z being equal to $\overline{VZ}$ and v being the height of the valve. In this case a system of points, (fig. 4), through which a single straight line cannot be drawn, is also obtained. As in the preceding case, here too three straight lines can be fixed at the same intervals. The statistical comparison of these three series can be carried out on the precondition that each of them is taken for an individual sample (tab. 3a, b, c). In comparing the differences between the regression coefficients of each two samples and the determinative deviation of the correlation coefficients, it was found that the differences between individual samples are statistically significant. (In the sample 1—7 and 7—18 $\frac{db}{\sigma_{db}} = 13{,}68$;

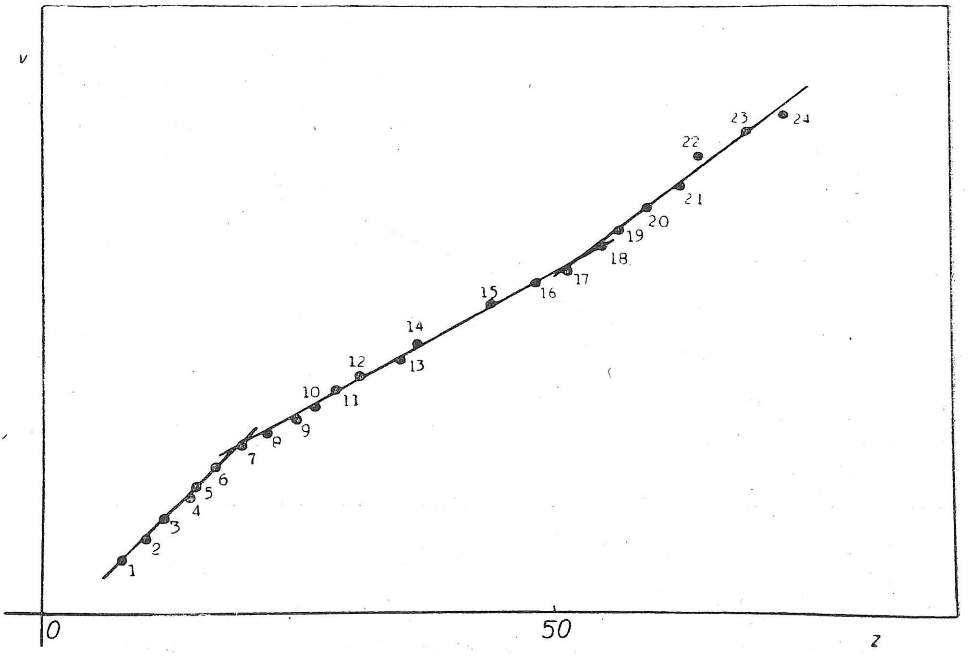

Fig. 4

in the samples 7—18 and 18—24 $\frac{db}{\sigma_{dq}} = 7{,}33$, in the samples 1—7 and 18—24 $\frac{db}{\sigma_{db}} = 5{,}24$. Consequently, the same conclusion as in the preceding case can be drawn.

Further, the growth regularity of the individual ontogenetic stages was studied in relation to the height and length of the valve. An arbitrary length of growth processes of a specimen divided into 24 intervals, which represent the succesive growth degrees, was taken as the basis. When evaluating the increase in the height according to the individual growth degrees, the course of the A polygons of the heights can be rather accurately substituted by the straight line B (fig. 5), the slope of which has the value of $k = 0{,}182$ and the intercept on Y-axis, chosen by the fourth growth degree, $q = 11{,}0$. Consequently, we obtain a straight line, the equation of which is:

$$y = 0{,}182x + 11 \text{ for } 1 \leqq \times \leqq 24$$

A greater deviation of the A polygon of the heights from the course of the substituting straight line B exists in the interval between the tenth and the fourteenth growth degree. However, it is typical of this interval as an intercept that its slope $k' = 0{,}176$, which is consequently very close to the slope of the substituting straight line. From this it is evident that the accuracy of the course of the substituting straight line is only slightly changed by the mentioned deviation.

When evaluating the increase of the length in individual growth degrees we can rather accurately substitute the polygon C of the lengths by the exponential function D (fig. 5), the common formula of which is

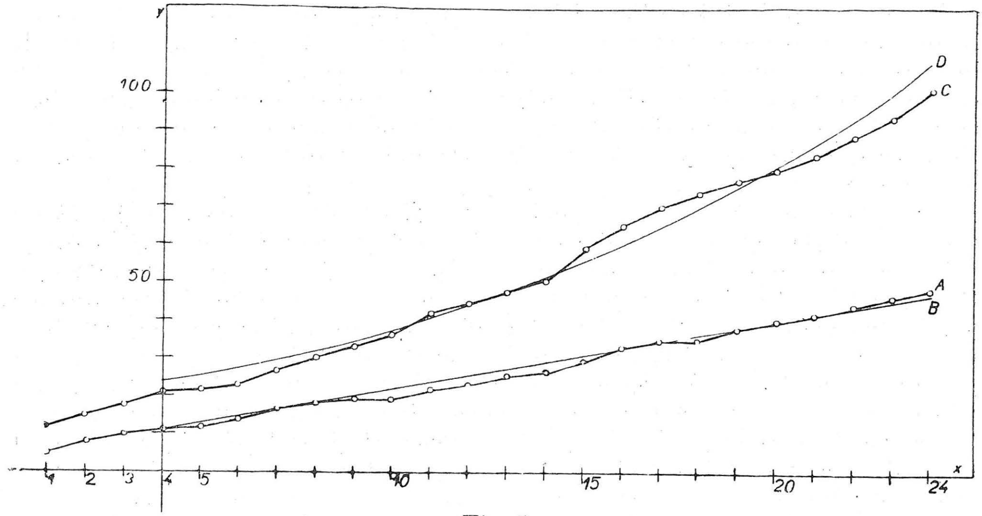

Fig. 5

$$y = a \,.\, e^{\alpha x} \tag{1}$$

where a, α are the convenient constants. In order to determine these constants equation (1) will be modified as follows:

$$\log y = \log a + \alpha \,.\, x \,.\, \log e \tag{2}$$

Consequently, after the logarithmic calculation of equation (1) a straight line is obtained which does not pass through the origin and its graph is then constructed on corresponding graphical paper (fig. 6).

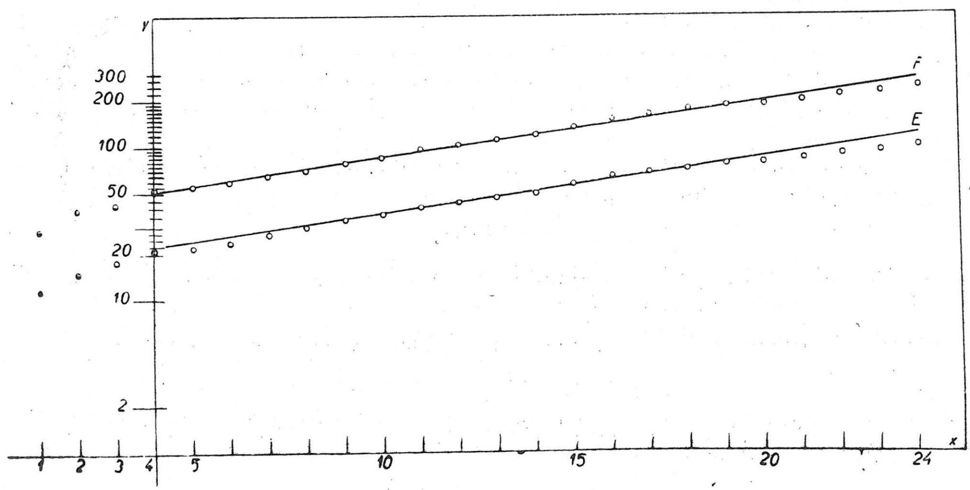

Fig. 6

As becomes evident from the original graph (fig. 5) as well as from the mentioned representation of the exponential function by the straight line E (fig. 6), the length increases in the course of the growth degrees 1—3 more slowly than it is shown by the course of the exponential function. In the growth degrees 15—19, however, a certain small difference in the growth appears, the length increasing somewhat more rapidly than according to the exponential function (1), in the growth degrees 21—24 again a little more slowly.

These differences can be in our opinion explained in the following way: If we suppose that the animal under study lived throughout its life in the same ecological conditions, we can explain the mentioned definitions of the growth of the length so that in the first to third growth degree (or even further, in the fourth to sixth) the genetic tendency to form a rostrum did not yet appear in the protoconchal stage of the valve, consequently, these growth degrees still appear generically and specifically indifferent. A evidence of a systematical position appears only when the prerostral stage is over and the genetic tendency to form a rostrum is evident. From this moment the time of adolescence sets in, demonstrating itself in the rapid development of the typical shape of the valve, which is probably coincident with the sexual maturity. This appears in our graph as a second difference (which means that the length increase is more rapid than the course of the theoretical exponential function). This growth stage seems to terminate in the eighteenth growth degree. From this degree the growth of the length shows a decreasing tendency, the course of the polygon intersecting the substituting exponential function approximately in the twentieth growth degree, after which the growth of the length continues to proceed more slowly than is supposed by the substituting exponential function. This last difference is in our opinion due to the passage of the individual into the gerontic stage. It is interesting that the division of the sequence of 24 growth degrees into 4 stages became apparent also in the study of v/d and v/z ratio by means of the regression lines with rather corresponding boundaries.

The difference on the course of the two relations C and D in the fourth, fifth and sixth growth degree can be neglected, as the connecting line of the degrees 4, 5 and 6 has the same curvature as the exponential function (1) which runs for these values of the independent variable x.

In all remaining growth degrees the length increases rather accurately according to the exponential function (1), so that the interval from the fourth to twenty fourth degree can be taken for the basis, being very suitable for the appreciation of this growth.

The slope of the straight line E which represents the exponential function (1) on the corresponding graphical paper has the value

$$k = 0{,}134.$$

For the intercept q of the straight line E on the Y-axis chosen by the fourth growth degree the value $q = 21{,}5$ is obtained.

In order to calculate the constants a, α of equation (1) let us denote

$$\eta = m_2 \,.\, \log y, \quad \xi = m_1 \,.\, x,$$

where m_1, m_2 are the scale coefficients of the graphical papers; by substitution into equation (2) we get

$$\frac{\eta}{m_2} = \alpha \,.\, \log e \,.\, \frac{\xi}{m_1} + \log a,$$

$$\eta = \frac{m_2}{m_1} . \alpha . \log e . \xi + m_2 . \log a,$$

consequently the equation of a straight line:

$$\eta = k . \xi + q,$$

where
$$k = \frac{m_2}{m_1} . \alpha . \log e = 0{,}134, \tag{3}$$
$$q = m_2 \log a = 21{,}5. \tag{4}$$

From equation (3) we obtain

$$\alpha = \frac{0{,}134}{40 . 0{,}434} = 0{,}0077,$$

from equation (4) we receive

$$a = 3{,}45.$$

Owing to the basic lengths of the intervals of individual growth degrees being selected for the graphical representation on fig. 5 quite arbitrarily, it is necessary to coordinate the sizes of the units of the independent variable values of x with the values of the function (in our case a given empirical function) by means of the coefficient m_3 by which the constant a is multiplied; in our case $m_3 = 5$ is convenient. This will be ascertained e. g. by the following way: the mean value of the functional values is $\bar{y} = 51$; the size of the length in the fourteenth growth degree correponds to this value. The functional value of the exponential function

$$y = 3{,}45 . e^{0{,}0077x},$$

if $x = 14$ then $y = 10{,}1$, therefore the ratio of the sizes of the units is really $m_3 = 5$.

The exponential function (1) receives consequently its final expression

$$y = 17{,}25 . e^{0{,}0077x} \quad \text{for } 4 \leqq x \leqq 24. \tag{5}$$

The graphical representation of the H function (5), which now also means the ratio of the sizes of the units of the growth degrees and of the sizes of the lengths, is reproduced in fig. 7.

In order that the constant $a = 17{,}25$ beused in equation (5) the intercept of the straight line q, which on the corresponding graphical paper represents function (5), must according to equation (4) equal:

$$q = m_2 . \log a = 40 . 1{,}2367 = 49{,}5$$

Now the polygon G is constructed from the sizes of the lengths in the scale adapted by the coefficient $m_3 = 5$ (fig.7) and at the same time exponential function (5) is converted into a straight line (straight line F on fig. 6). We will find that the straight line F runs parallel with the straight line E, representing the basic exponential function, consequently both straight lines have the same slope; straight line F intersects the Y-axis at the end point of the intercept q; by measuring it is found that this intercept is approximately equal to the computed intercept. Therefore, the value of the coefficient $m_3 = 5$ is right.

Owing to what was ascertained above it can be stated that while the increase of the heights v in the individual growth degrees proceeds in an arithmetical progression, as the curve expressing this development is a straight line, the increase of the

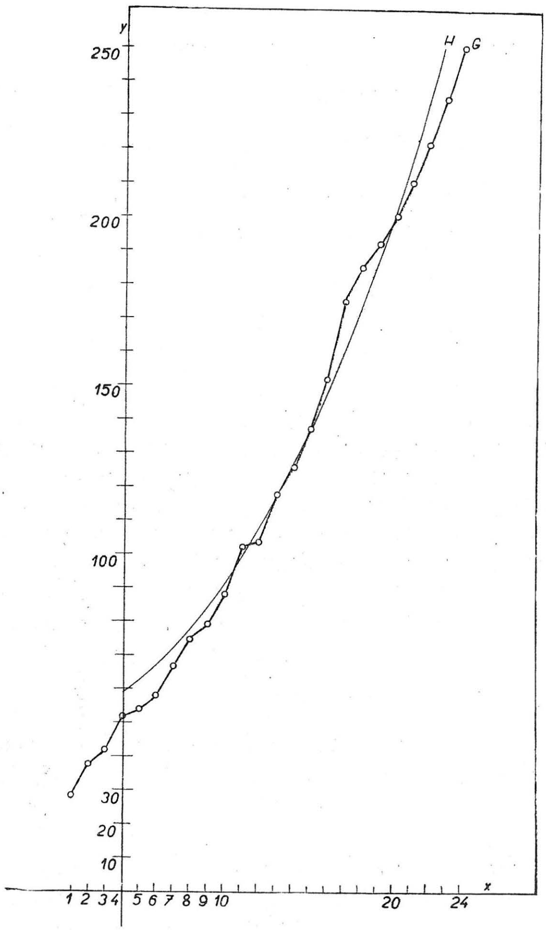

Fig. 7

lengths d proceeds in a geometrical progression, because the curve expressing this relation is an exponential function, and the values x of the growth degrees proceed in an arithmetical series. This can be generally expressed as follows:

Let the value of the height v in the first growth degree be denoted x_1, the corresponding value of the length d in the first growth degree d_1.

Then the arithmetical progression of the growth degrees is

$$x = x_1, \quad x_1 + c, \quad x_1 + 2c, \ldots\ldots\ldots x_1 + n \,.\, c$$

where c is the difference of the progression. From the equation

$$x = x_1 + n \,.\, c$$

we obtain

$$n = \frac{x - x_1}{c} \,. \tag{6}$$

The exponential function will be expressed by the simplest general form:

$$d = A \cdot q^x . \tag{7}$$

On the assumption that the lengths d_n increase according to a geometrical progression, the individual values of the lengths form the progression

$$d = d_1 , \quad d_1 \cdot p , \quad d_1 \cdot p^2 , \ldots\ldots\ldots d_1 \cdot p^n ,$$

where p is the quotient of the progression. If we introduce equation (6) into the formula

$$d = d_1 \cdot p^n$$

we get

$$d = d_1 \cdot p^{-\frac{x1}{c}} \cdot \left(p^{\frac{1}{c}}\right)^x ;$$

if we lay

$$d_1 \cdot p^{-\frac{x1}{c}} = A , \quad p^{\frac{1}{c}} = q$$

we do get the exponential function (7).

In the course of a continuous growth in a homogenous environment the increase of the height would proceed in an arithmetical series with the difference $r = 1.88$, where r is the mean value of all differences which vary within the interval

$$0{,}7 \leqq \mathrm{r} \leqq 4{,}0$$

This progression P_v is presented on pl. 4 for the growth degrees $n = 1$ to $n = 24$ and compared with the measured values of the heights (in the third row). The progression P_v is arranged according to the last member.

Under the above mentioned conditions of the growth the increase of the length d would proceed in a geometrical progression with a quotient $q = 1.08$, where q is the mean value of all quotients which vary within the interval (the initial three degrees were not taken in account)

$$1{,}02 \leqq q \leqq 1{,}16$$

This progression of the lengths P_d is demonstrated on table 5 for $n = 4$ to $n = 24$. In the third row of the table are the measured values of the lengths d, in the fourth row the values F_d of the substituting exponential function (5) and in the second row the geometrical progression of the lengths P^d. The progression P_d is arranged according to the member for $n = 11$, because the measured length d_{11} is here nearly identical with the length computed from the substituting function (5) for $n = 11$.

The course of the curve D on fig. 5 and of the curve H on fig. 7 is rather similar to the general parabolic function of the type

$$y = a \cdot x^n .$$

However, the increase of the lengths in the individual growth degrees does not proceed according to the parabolic function, as after the logarithmic calculation of this function we obtain

$$\log y = \log a + n \cdot \log x ,$$

consequently a straight line of the expression

$$\eta = n \,.\, \xi + \log a\,,$$

which does not agree with the graphical representation of the values of the length of the growth degrees presented on the corresponding graphical paper (fig. 8).

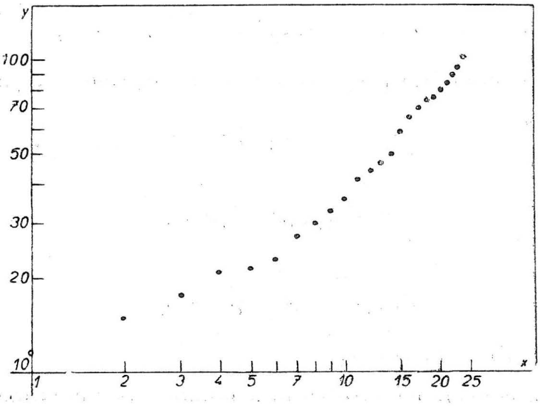

Fig. 8

Likewise, the parabolae of the type

$$y = a \,.\, (x - x_o)^n$$

may be excluded.

Conclusion

To date it cannot be stated as to what extent the different characters of the growth of the lengths and heights of nuculanid shells can serve as a guide for specific differenciation. Only further studies will prove how this character can be applied in the biometrical studies of the nuculanid assemblages.

REFERENCES

KUMPERA, O.—PRANTL, F.—RŮŽIČKA, B. (1960): Revision of the Nuculanidae from the Ostrava-Karviná District (Pelecypoda).—Acta Mus. Nationalis Pragae, sv. XVI, č. 1/2.

RŮŽIČKA, B.—BOJKOWSKI, J. (1960): Polidevcia hrebnickii, nov. spec., New Pelecypod from the Lower Ostrava Beds.—Přírodovědný časopis slezský, r. XXI, č. 4.

Table I

n	l	v	ξ	η	$\xi \cdot \eta$	ξ^2	η^2
1	14,40	5,40	43,92	20,68	908,26	1 928,96	427,66
2	18,70	7,30	39,62	18,78	744,06	1 569,74	352,68
3	21,90	9,50	36,42	16,58	603,84	1 326,41	274,89
4	25,90	11,60	32,42	14,48	469,44	1 051,05	209,67
5	28,00	12,50	30,32	13,58	411,74	919,30	184,41
6	31,00	14,50	27,32	11,58	316,36	746,38	134,09
7	35,50	16,80	22,82	9,28	211,76	520,75	86,11
8	38,50	17,50	19,82	8,58	170,05	392,83	73,61
9	41,90	19,00	16,42	7,08	116,25	269,61	50,12
10	42,80	20,50	15,52	5,58	86,60	240,87	31,13
11	47,70	22,00	10,62	4,08	43,32	112,78	16,64
12	52,00	23,50	6,32	2,58	16,30	39,94	6,65
13	56,50	25,00	1,82	1,08	1,96	3,31	1,16
14	59,00	26,50	0,68	0,42	0,28	0,46	0,17
15	67,50	30,50	9,18	4,42	40,57	84,27	19,53
16	73,90	32,50	15,58	6,42	100,02	242,73	42,21
17	78,40	34,00	20,08	7,92	139,03	403,20	62,72
18	82,50	36,00	24,18	9,92	239,86	584,67	98,40
19	85,20	38,00	26,88	11,92	320,40	722,53	142,08
20	89,00	40,00	30,68	13,92	427,06	941,26	193,76
21	94,10	42,00	35,78	15,92	569,61	1 280,20	253,44
22	99,40	45,00	41,08	18,92	777,23	1 687,56	357,96
23	104,00	47,40	45,68	21,32	973,89	2 086,66	454,54
24	112,00	49,00	53,68	22,92	1 230,34	2 881,54	525,32
Σ	1 399,80	626,00			8 938,23	20 037,01	3 997,95
$\frac{\Sigma}{n} = A$	58,32	26,08				834,87	166,58

$\sigma_l = 28{,}88$; $\sigma_a = 12{,}90$

$K_{lv} = 0{,}999$

$b_{vl} = 2{,}216$

$b_{lv} = 0{,}442$

$v = 0{,}442 . l + 0{,}42$

$l = 2{,}216\, v + 0{,}43$

Table 2a

n	d	v	ξ	η	$\xi \cdot \eta$	ξ^2	η^2
1	12,50	5,40	7,70	5,68	43,73	59,29	32,26
2	15,80	7,30	4,40	3,78	16,63	19,36	14,28
3	17,80	9,50	2,40	1,58	3,79	5,76	2,49
4	21,50	11,60	1,30	0,52	0,67	1,69	0,27
5	22,00	12,50	1,80	1,42	2,55	3,24	2,01
6	24,00	14,50	3,80	3,42	12,99	14,44	11,69
7	27,80	16,80	7,60	5,72	43,47	57,76	32,71
Σ	141,40	77,60			123,83	161,54	95,71
$\frac{\Sigma}{n} = A$	20,20	11,08				23,07	13,67

$\sigma_d = 4{,}804$; $\sigma_v = 3{,}697$

$K_{dv} = 0{,}996$

$$v = 0{,}766 \,.\, d - 4{,}39$$

$b_{vd} = 1{,}294$

$b_{dv} = 0{,}766$

Table 2b

n	d	v	ξ	η	$\xi \cdot \eta$	ξ^2	η^2
1	27,80	16,80	20,82	8,51	177,17	433,47	72,42
2	31,00	17,50	17,62	7,81	137,61	310,46	60,99
3	33,00	19,00	15,62	6,31	98,56	243,98	39,81
4	36,20	20,50	12,42	4,81	59,74	154,25	23,13
5	40,50	22,00	8,12	3,31	26,87	65,92	10,95
6	43,90	23,50	4,72	1,81	8,54	22,27	3,27
7	48,80	25,00	0,18	0,31	0,05	0,03	0,09
8	51,50	26,50	2,88	1,19	3,42	8,29	1,41
9	60,00	30,50	11,38	5,19	59,06	129,50	26,93
10	65,80	32,50	17,18	7,19	123,52	295,15	51,69
11	70,00	34,00	21,38	8,69	185,79	457,10	75,51
12	75,00	36,00	26,38	10,69	282,00	695,90	114,27
Σ	583,50	303,80			1 162,33	2 816,33	480,47
$\frac{\Sigma}{n} = A$	48,62	25,31				234,69	40,03

$\sigma_d = 15{,}320$; $\sigma_v = 6{,}326$

$K_{dv} = 0{,}999$

$b_{vd} = 2{,}419$ $\qquad v = 0{,}412 \,.\, d + 5{,}28$

$b_{dv} = 0{,}412$

Table 2c

n	d	v	ξ	η	$\xi \cdot \eta$	ξ^2	η^2
1	75,00	36,00	11,00	6,48	71,28	121,00	41,99
2	76,50	38,00	9,50	4,48	42,56	90,25	20,07
3	81,00	40,00	5,00	2,48	12,40	25,00	6,15
4	85,00	42,00	1,00	0,48	0,48	1,00	0,23
5	90,00	45,00	4,00	2,52	10,08	16,00	6,35
6	94,00	47,40	8,00	4,92	39,36	64,00	24,20
7	100,50	49,00	14,50	6,52	94,54	210,25	42,51
Σ	602,00	297,40			270,70	527,50	141,50
$\frac{\Sigma}{n} = A$	86,00	42,48				75,35	20,21

$\sigma_d = 8{,}680$; $\sigma_v = 4{,}496$

$K_{dv} = 0{,}991$

$b_{vd} = 1{,}913$ $\qquad\qquad v = 0{,}513 \,.\, d - 1{,}64$

$b_{dv} = 0{,}513$

Table 3a

n	z	v	ξ	η	$\xi \cdot \eta$	ξ^2	η^2
1	7,90	5,40	5,88	5,68	33,39	34,57	32,26
2	10,20	7,30	3,58	3,78	13,53	12,81	14,28
3	12,00	9,50	1,78	1,58	2,81	3,16	2,49
4	14,50	11,60	0,72	0,52	0,37	0,51	0,27
5	15,10	12,50	1,32	1,42	1,87	1,74	2,01
6	17,00	14,50	3,22	3,42	11,01	10,36	11,69
7	19,18	16,80	6,02	5,72	34,43	36,24	32,71
Σ	96,50	77,60			97,41	99,39	95,71
$\frac{\Sigma}{n} = A$	13,78	11,08				14,19	13,67

$\sigma_z = 3{,}766$; $\sigma_v = 3{,}697$

$K_{zv} = 0{,}999$

$v = 0{,}980 \,.\, z - 2{,}42$

$b_{vz} = 1{,}017$

$z = 1{,}017 \,.\, v + 2{,}51$

$b_{zv} = 0{,}980$

Table 3b

n	z	v	ξ	η	$\xi . \eta$	ξ^2	η^2
1	19,80	16,80	15,46	8,51	131,56	239,01	72,42
2	22,00	17,50	13,26	7,81	103,56	175,82	60,99
3	24,90	19,00	10,36	6,31	65,37	107,32	39,81
4	26,60	20,50	8,66	4,81	41,65	74,99	23,13
5	28,50	22,00	6,76	3,31	22,37	45,69	10,95
6	31,00	23,50	4,26	1,81	7,71	18,14	3,27
7	34,50	25,00	0,76	0,31	0,23	0,57	0,09
8	37,00	26,50	1,74	1,19	2,07	3,02	1,41
9	44,00	30,50	8,74	5,19	45,36	76,38	26,93
10	48,50	32,50	13,24	7,19	95,19	175,29	51,69
11	51,90	34,00	16,64	8,69	144,60	276,88	75,51
12	54,50	36,00	19,24	10,69	205,67	370,17	114,27
Σ	423,20	303,80			865,34	1 563,28	480,47
$\frac{\Sigma}{n} = A$	35,26	25,31				130,27	40,03

$\sigma_z = 11{,}413$; $\sigma_v = 6{,}326$

$K_{zd} = 0{,}998$

$b_{vz} = 1{,}800$ $\qquad v = 0{,}533 . d + 5{,}82$

$b_{zv} = 0{,}553$

Table 3c

n	z	v	ξ	η	$\xi . \eta$	ξ^2	η^2
1	54,50	36,00	7,87	6,48	50,99	61,93	41,99
2	56,40	38,00	5,97	4,48	26,74	35,64	20,07
3	59,00	40,00	3,37	2,48	8,35	11,35	6,15
4	62,00	42,00	0,37	0,48	0,17	0,13	0,23
5	64,50	45,00	2,13	2,52	5,36	4,53	6,35
6	68,20	47,40	5,83	4,92	28,68	33,98	24,20
7	72,00	49,00	9,63	6,52	62,78	92,73	42,51
Σ	436,60	297,40			183,07	240,29	141,50
$\frac{\Sigma}{n} = A$	62,37	42,48				34,32	20,21

$\sigma_z = 5{,}859$; $\sigma_v = 4{,}496$

$K_{zv} = 0{,}993$

$b_{vz} = 1{,}294$

$v = 0{,}762 . z — 5{,}04$

$b_{zv} = 0{,}762$

Table 4

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Pv	5,76	7,64	9,52	11,40	13,28	15,66	17,04	18,92	20,80	24,56	22,68	26,44	28,32	30,20	32,08	33,96	35,84	37,72	39,60	41,48	43,36	45,24	47,12	49,00
v	5,4	7,3	9,5	11,6	12,5	14,5	16,8	17,5	19,0	20,5	22,0	23,5	25,0	26,5	30,5	32,5	34,0	36,0	38,0	40,0	42,0	45,0	47,4	49,0

Table 5

n	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Pd	23,63	25,52	27,56	29,77	32,15	34,72	37,50	40,50	43,74	47,23	51,00	55,08	59,48	64,23	69,36	74,90	80,89	87,36	94,34	101,88	110,03
d	21,5	22,0	24,0	27,8	31,0	33,0	36,2	40,5	43,9	48,8	51,5	60,0	65,8	70,0	75,0	76,5	81,0	85,0	90,0	94,0	100,5
Fd	23,4	25,3	27,4	29,6	32,0	34,5	37,3	40,3	43,5	47,0	50,8	54,9	59,2	64,0	69,1	74,7	80,6	87,0	94,0	101,0	109,5