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Biometrická studie obrysové křivky některých mytiloidních mlžů (Pelecypoda)

Biometrical Study of the Outline of Some Mytiloid Pelecypods

(S 8 tabulkami a 1 obrázkem v textu)

(With 8 tables and 1 text figure)

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Úkolem této práce bylo zjištění, do jaké míry by biometrická studia obrysových křivek mlžů, které J. Barrande (1881) řadil k rodu *Mytilus*, mohla být vodítkem při rozlišení jednotlivých Barrandem stanovených druhů. Práce sama prokázala, že toto allochronní i allopatrické společenstvo nelze vzájemně rozlišit na základě biometrických studií obrysových křivek, protože tyto křivky jsou si tak blízké, že nevykazují odlišnost, která by měla statistický význam. Z tohoto důvodu druhové rozlišení výše zmíněných forem by mohlo podle našeho názoru být založeno pouze na vnitřní charakteristice (zámkový aparát, svalové vtisky atd.), která však u žádného z Barrandových jedinců není patrná. Dále není vyloučeno, že většina mytiloidních rodů má stavbu obrysové křivky v základních rysech tak blízkou, že nebude možno tohoto znaku využít k druhovému rozlišování pomocí biometrické analýzy tam, kde vnitřní systematické znaky nejsou známy nebo studovatelné.

In the course of a systematical revision of the pelecypods of Barrande's collection, which we have carried out for several years, we have received the task of studying the forms which J. Barrande (1881) designated by the cumulative name *Mytilus* (non *Mytilus* Linné, 1758). Besides Barrande's material a number of specimens coming from recent collections, especially from those of J. Kříž, were also included into the study. However, none of the studied specimens showed any traces of the hinge apparatus, muscle scars or pallial line. The surface sculpture, as far as it is preserved in individual cases, shows various character due to the fossilization of different layers of the valve matter. The present study was worked up in order to find out how much the external habitus, expressed primarily by the outline of the valve, can be significant in the specific differenciation. The reconstructed and ten times enlarged outlines were used as a basis, ninety-two of them being taken from Barrande's original material, the rest from recent collections. It should be noted that all this material was treated as a whole, although the individual specimens of this assemblage came from different stratigraphical levels. In this way we tried to get already at the beginning of our analysis a knowledge of whether the specimens of one stratigraphical level will constitute assemblages significantly different from those of another stratigraphical position.

On the reconstructed outlines the basic parameters, which in our opinion fully express the character of the valve, were determined (see fig. 1). The individual parameters (characters) were denoted as follows:

$\overline{OC} = \overline{OD} = \overline{OB} = a$, $\overline{AE} = b$, $m + m' = c$, $\overline{AD} = d$, $\overline{BC} = e$ $\overline{AC} = m$, $\overline{AB} = m'$

	a	b	c	d	e	m	m'	$(a_i-\bar{a})^2$	$(b_i-\bar{b})^2$	$(c_i-\bar{c})^2$	$(d_i-\bar{d})^2$	$(e_i-\bar{e})^2$	$(a_i-\bar{a})$	$(b_i-\bar{b})$	$(c_i-\bar{c})$	$(d_i-\bar{d})$	$(e_i-\bar{e})$	$(a_i-\bar{a})(b_i-\bar{b})$	$(a_i-\bar{a})(c_i-\bar{c})$	$(a_i-\bar{a})(d_i-\bar{d})$	$(a_i-\bar{a})(e_i-\bar{e})$	$(b_i-\bar{b})(c_i-\bar{c})$	$(b_i-\bar{b})(d_i-\bar{d})$	$(c_i-\bar{c})(d_i-\bar{d})$
1	13	38	84	54	35	40	44	357,2	404,0	2265,7	1568,2	951,1	- 18,9	- 20,1	- 47,6	- 39,6	- 30,8	379,9	899,6	748,4	582,1	956,76	795,96	1884,96
2	21	42	98	65	50	50	48	118,8	259,2	1129,6	818,0	250,9	- 10,9	- 16,1	- 33,6	- 28,6	- 15,8	175,5	366,2	3135,8	172,2	540,96	460,46	960,96
3	35	54	128	87	69	65	63	9,6	16,8	13,0	43,6	10,0	3,1	- 4,1	- 3,6	- 6,6	3,2	- 12,7	- 11,2	20,5	9,9	14,76	27,06	23,76
4	36	68	152	118	67	77	75	16,8	98,0	415,7	595,4	1,3	4,1	9,9	20,4	24,4	1,2	40,6	83,6	100,0	4,9	210,96	241,56	497,76
5	53	91	211	170	109	98	113	445,2	1082,4	6302,8	5837,0	1862,8	21,1	32,9	79,4	76,4	43,2	694,2	1675,3	1612,0	911,5	2612,26	2513,56	6066,16
6	75	112	266	190	148	134	132	1857,6	2905,2	18060,7	9293,0	6750,3	43,1	53,9	134,4	96,4	82,2	2323,1	5792,6	4154,8	3542,8	7244,16	5195,96	12956,16
7	41	60	143	110	79	71	72	82,8	3,6	129,7	269,0	186,3	9,1	1,9	11,4	16,4	13,2	17,3	103,7	148,2	120,1	21,66	31,16	186,96
8	54	86	200	158	103	102	98	488,4	778,4	4745,6	4147,4	1418,0	22,1	27,9	68,4	64,4	37,2	616,6	1511,6	1423,2	822,1	1908,36	1796,76	4404,96
9	54	85	194	143	107	103	91	488,4	723,6	3892,5	2440,4	1694,1	22,1	26,9	62,4	49,4	41,2	594,5	1379,0	1091,7	910,5	1678,56	1328,86	3082,56
10	48	77	179	134	93	89	90	259,2	357,2	2245,8	1632,2	737,7	16,1	18,9	47,4	40,4	27,2	304,3	763,1	650,4	437,9	895,86	763,56	1914,96
11	62	91	219	170	120	108	111	906,0	1082,4	7637,0	5837,0	2933,3	40,1	32,9	87,4	76,4	54,2	1319,3	3504,7	3063,6	2173,4	2875,46	2513,56	6677,36
12	42	73	167	113	83	84	83	102,0	222,0	1252,4	376,4	294,5	10,1	14,9	35,4	19,4	17,2	150,5	357,5	195,9	173,7	527,46	289,06	686,76
13	43	93	205	131	85	103	102	123,2	1218,0	5386,1	1398,8	367,1	11,1	34,9	73,4	37,4	19,2	387,4	814,7	415,1	213,1	2561,66	1305,26	2745,16
14	13	39	84	54	35	40	44	357,2	383,9	2266,7	1568,2	948,6	- 18,9	- 19,1	- 47,6	- 39,6	- 30,8	361,0	899,6	748,4	582,1	909,16	756,36	1884,96
15	31	71	156	118	68	75	81	1,0	166,4	594,9	595,4	4,8	- 0,9	12,9	24,4	24,4	2,2	- 11,6	- 22,0	- 22,0	- 2,0	314,76	314,76	595,36
16	26	39	91	65	50	45	46	34,8	364,8	1649,2	818,0	249,6	- 5,9	- 19,1	- 40,6	- 28,6	- 15,8	112,7	239,5	168,7	93,2	775,46	546,26	1161,16
17	21	58	124	80	50	58	66	118,8	0,0	57,9	185,0	249,6	- 10,9	- 0,1	- 7,6	- 13,6	- 15,8	1,1	82,8	148,2	172,2	0,76	1,36	103,36
18	50	68	167	117	100	86	81	327,6	98,0	1252,4	547,6	1169,6	18,1	9,9	35,4	23,4	34,2	179,2	640,7	423,5	619,0	350,46	231,66	828,36
19	35	53	127	85	69	64	63	9,6	26,0	21,2	74,0	10,2	3,1	- 5,1	- 4,6	- 8,6	3,2	- 15,8	- 14,3	- 26,7	9,9	23,46	43,86	39,56
20	34	72	156	107	65	74	82	4,4	193,2	549,9	179,6	0,6	2,1	13,9	24,4	13,4	- 0,8	29,2	51,2	28,1	- 1,7	339,16	186,26	326,96
21	18	39	85	54	35	44	41	193,2	383,9	2219,1	1568,2	951,1	- 13,9	- 19,1	- 47,6	- 39,6	- 30,8	265,5	661,6	550,1	428,1	909,16	756,36	1884,96
22	31	49	113	84	59	56	57	1,0	91,9	346,3	92,2	46,2	- 0,9	- 9,1	- 18,6	- 9,6	- 6,8	8,2	16,7	8,6	6,1	169,26	87,36	178,56
23	36	64	135	102	71	62	73	16,8	34,8	11,5	70,6	27,0	4,1	5,9	3,4	8,4	5,2	24,2	13,9	34,4	21,3	20,06	49,56	28,56
24	29	47	109	74	57	55	54	8,4	123,2	511,2	384,2	77,4	- 2,9	- 11,1	- 22,6	- 19,6	- 8,8	32,2	65,5	56,9	24,6	250,86	217,56	442,96
25	43	70	158	112	85	73	85	123,2	141,6	696,4	338,6	367,1	12,1	11,9	26,4	18,4	19,2	144,0	319,4	222,4	232,3	314,16	218,96	485,76
26	24	37	89	63	50	45	44	62,4	445,2	1815,6	936,4	249,6	- 7,9	- 21,1	- 42,6	- 30,6	- 15,8	166,7	336,5	241,7	124,8	898,86	645,66	1303,56
27	26	41	91	70	50	44	47	34,8	292,4	1649,1	557,0	249,6	- 5,9	- 17,1	- 40,6	- 23,6	- 15,8	100,9	239,5	139,2	93,2	694,26	403,56	958,16
28	11	29	65	45	32	31	34	436,8	846,8	4436,9	2362,0	1142,4	- 20,9	- 29,1	- 66,6	- 48,6	- 33,8	508,2	1391,9	1015,7	706,4	1938,06	1414,26	3236,76
29	14	25	57	41	27	28	29	320,4	1095,6	5566,6	2766,8	1505,4	- 17,9	- 33,1	- 74,6	- 52,6	- 38,8	592,5	1335,3	941,5	694,5	2469,26	1741,06	3923,96
30	24	44	97	71	47	44	53	62,4	198,8	1197,8	510,8	353,4	- 7,9	- 14,1	- 34,6	- 22,6	- 18,8	111,4	287,6	178,5	148,5	487,86	318,66	781,96
31	24	40	91	66	47	49	42	62,4	327,6	1649,1	761,8	353,4	- 7,9	- 18,1	- 40,6	- 27,6	- 18,8	143,0	320,7	218,0	148,5	734,86	517,66	1120,56
32	25	50	109	77	48	63	46	47,4	65,6	511,2	275,6	316,8	- 6,9	- 8,1	- 22,6	- 16,6	- 17,8	55,9	155,9	114,5	122,8	183,06	134,46	375,16
33	17	28	65	45	33	34	31	222,0	906,0	4436,9	2362,0	1075,8	- 14,9	- 30,1	- 66,6	- 48,6	- 32,8	448,5	992,g	724,1	488,7	2004,66	1462,86	3236,76
34	40	65	150	106	79	77	73	65,6	47,6	338,2	153,8	174,2	8,1	6,9	18,4	12,4	13,2	55,9	149,0	100,7	106,9	126,96	85,56	228,16
35	35	54	124	94	65	64	60	9,6	16,8	57,9	0,2	0,8	3,1	- 4,1	- 7,6	0,4	- 0,8	- 12,7	- 23,6	1,2	- 2,5	31,16	- 1,64	- 3,00
36	15	29	66	44	31	34	32	285,6	846,8	4304,7	2460,2	1211,0	- 16,9	- 29,1	- 65,6	- 49,6	- 34,8	491,8	1108,6	838,2	588,1	1908,96	1443,36	3253,76
37	42	88	182	125	83	100	82	102,0	894,0	2539,1	986,0	294,5	10,9	29,9	50,4	31,4	17,2	302,0	509,0	317,2	173,7	1506,96	938,86	1582,56
38	30	63	139	98	58	71	68	3,6	24,0	54,6	19,4	60,8	- 1,9	4,9	7,4	4,4	- 7,8	- 9,3	- 14,1	- 8,4	14,8	36,26	21,56	32,56
39	34	70	159	113	77	82	77	4,4	141,6	750,2	376,4	125,4	2,1	11,9	27,4	19,4	11,2	25,0	57,5	40,7	23,5	326,06	230,86	531,56
40	26	51	114	83	48	60	54	34,8	50,4	310,1	112,4	316,8	- 5,9	- 7,1	- 17,6	- 10,6	- 17,8	41,9	103,8	62,5	105,0	124,96	75,26	186,56
41	20	32	85	59	40	42	43	141,6	681,2	2219,1	1197,2	665,6	- 11,9	- 26,1	- 46,6	- 34,6	- 25,8	310,6	554,5	411,7	307,0	1216,26	903,06	1612,36
42	31	74	162	110	61	82	80	1,0	252,8	923,5	269,0	23,0	- 0,9	15,9	30,4	16,4	- 4,8	- 14,3	- 27,4	- 14,8	4,3	483,36	260,76	498,56
43	21	39	88	62	42	45	43	118,8	364,8	1901,8	998,6	566,4	- 10,9	- 19,1	- 43,6	- 31,6	- 23,8	208,2	475,2	344,4	259,4	832,76	603,56	1377,76
44	27	43	101	69	52	51	50	24,0	228,0	937,0	605,2	190,4	- 4,9	- 15,1	- 30,6	- 24,6	- 13,8	74,0	149,9	120,5	67,6	462,06	317,46	752,76
45	23	41	93	66	45	47	46	79,2	292,4	1490,7	761,8	432,6	- 8,9	- 17,1	- 38,6	- 27,6	- 20,8	152,2	343,5	245,6	185,1	660,06	471,96	1065,36
46	18	31	70	53	31	36	34	193,2	734,4	3795,8	1648,4	1211,0	- 13,9	- 27,1	- 61,6	- 40,6	- 34,8	376,7	856,2	564,3	483,7	1669,36	1100,26	2500,96
47	19	42	91	62	36	44	47	166,4	259,2	1649,1	998,6	888,0	- 12,9	- 16,1	- 40,6	- 31,6	- 29,8	207,7	523,7	407,6	384,4	653,66	508,76	1282,96
48	45	127	268	171	89	134	134	171,6	4747,2	18602,2	5990,8	538,2	13,1	68,9	136,4	77,4	23,2	902,6	1786,8	1013,9	303,9	9397,96	5332,86	10557,36
49	31	62	135	104	57	77	58	1,0	15,2	11,5	108,2	77,4	- 0,9	3,9	3,4	10,4	- 8,8	- 3,5	- 3,1	- 9,4	7,9	13,26	40,56	35,36
50	34	93	198	141	71	101	97	4,4	1218,0	4407,6	2246,8	27,0	2,1	34,9	66,4	47,4	5,2	73,3	139,4	99,5	10,9	2317,36	1654,26	3147,36
51	36	74	165	109	72	85	80	16,8	252,8	1114,9	237,2	38,4	4,1	15,9	33,4	15,4	6,2	65,2	136,9	63,1	25,4	531,06	244,86	514,36
52	28	55	127	89	53	61	66	15,2	9,6	21,2	21,2	163,8	- 3,9	- 3,1	- 4,6	- 4,6	- 12,8	12,1	17,9	17,9	49,9	14,26	14,26	21,16
53	36	66	148	103	72	82	66	16,8	62,4	268,6	88,4	38,4	4,1	7,9	16,4	9,4	6,2	32,4	67,2	38,5	25,4	129,56	74,26	154,16
54	42	75	169	109	81	92	74	102,0	285,6	1182,7	237,2	231,0	10,1	16,9	34,4	15,4	15,2	170,7	347,4	155,5	153,5	581,36	260,26	529,76
55	42	82	182	125	84	85	97	102,0	571,2	2539,1	986,0	331,2	10,1	23,9	50,4	31,4	18,2	241,4	579,0	317,5	183,8	1204,56	750,46	1582,56
56	39	86	178	114	77	81	97	50,4	778,4	2152,0	416,2	125,4	7,1	27,9	46,4	20,4	11,2	198,1	329,4	144,8	79,5	1294,56	569,16	946,56
57	21	37	85	65	41	41	44	118,8	445,2	2219,1	818,0	615,0	- 10,9	- 21,1	- 46,6	- 28,6	- 24,8	230,0	507,9	311,7	270,3	983,26	603,46	1332,96
58	25	37	90	63	50	45	45	47,4	445,2	1731,4	936,4	249,6	- 6,9	- 21,1	- 41,6	- 30,6	- 15,8	145,6	287,0	211,1	109,0	877,76	645,66	1272,96
59	42	69	161	110	84	79	82	102,0	118,8	863,8	269,0	331,2	10,1	10,9	29,4	16,4	18,2	110,1	296,9	165,6	183,8	320,46	178,76	482,16
60	42	78	174	125	83	84	90	102,0	396,0	1796,9	986,0	294,5	10,1	19,9	42,4	31,4	17,2	210,0	428,2	317,1	173,2	843,76	624,86	1331,36
61	26	41	95	73	49	47	48	34,8	292,4	1340,3	424,4	282,2	- 5,9	- 17,1	- 36,6	- 20,6	- 16,8	100,9	215,9	121,5	99,7	625,86	352,26	753,96
62	20	32	76	53	40	37	39	141,6	681,2	3092,5	1648,4	665,6	- 11,9	- 26,1	- 55,6	- 40,6	- 25,8	310,6	661,6	483,1	307,0	1451,16	1059,66	2257,36
63	34	61	137	103	65	72	65	4,4	8,4	29,0	88,4	0,6	2,1	2,9	5,4	9,4	- 0,8	6,1	11,3	19,7	- 1,7	15,66	27,26	50,76
64	17	34	75	52	32	36	39	222,0	580,8	3204,7	1730,6	1142,4	- 14,9	- 24,1	- 56,6	- 41,6	- 33,8	359,1	843,3	619,8	503,6	1364,06	1002,56	2354,56
65	31	52	119	84	60	62	57	1,0	37,2	159,0	92,2	33,6	- 0,9	- 6,1	- 12,6	- 9,6	- 5,8	5,5	11,3	8,6	5,2	76,86	58,56	120,96
66	30	49	114	83	59	54	60	3,6	82,8	310,1	112,4	46,2	- 1,9	- 9,1	- 17,6	- 10,6	- 6,8	17,3	33,4	20,1	12,9	160,16	96,46	186,56
67	55	84	198	156	105	104	94	533,6	670,8	4407,6	3893,8	1536,6	23,1	25,9	66,4	62,4	39,2	598,2	1533,8	1441,4	905,5	1719,76	1616,16	4143,36
68	50	101	223	163	96	115	108	327,6	1840,4	8352,1	4816,4	912,0	18,1	42,9	91,4	69,4	30,2	776,5	1654,3	1256,1	546,6	3921,06	2977,26	6343,16
69	28	44	101	72	55	57	44	15,2	198,8	937,0	466,6	116,6	- 3,9	- 14,1	- 30,6	- 21,4	- 10,8	55,0	119,3	84,2	42,1	431,46	304,56	660,96
70	37	81	178	114	72	89	89	26,0	524,4	2152,0	416,2	38,4	5,1	22,9	46,4	20,4	6,2	116,8	236,6	104,0	31,6	1062,56	467,16	946,56
71	33	56	125	86	67	53	72	1,2	4,4	43,7	57,8	1,4	1,1	- 2,1	- 6,6	- 7,6	1,2	- 2,3	- 7,3	- 8,4	1,3	13,86	15,96	50,16
72	61	120	269	184	120	144	125	846,8	3831,6	18876,0	8172,2	2937,6	29,1	61,9	137,4	90,4	54,2	1801,3	3998,3	2630,6	1577,2	8505,06	5595,76	12420,96
73	29	38	95	63	56	48	47	8,4	404,0	1340,3	936,4	96,0	- 2,9	- 20,1	- 36,6	- 30,6	- 9,8	58,3	106,1	88,7	28,4	735,66	615,06	1119,96
74	27	40	97	76	53	56	41	24,0	327,6	1197,9	309,8	163,8	- 4,9	- 18,1	- 34,6	- 17,6	- 12,8	88,7	169,5	86,2	62,7	626,26	318,56	608,96
75	24	38	88	68	45	47	41	62,4	404,0	1901,8	655,4	432,6	- 7,9	- 20,1	- 43,6	- 25,6	- 20,8	158,8	344,4	202,2	164,3	876,36	514,56	1116,16
76	12	29	63	44	24	31	32	396,0	846,8	4707,3	2460,2	1747,2	- 19,9	- 29,1	- 68,6	- 49,6	- 41,8	579,0	1365,1	987,0	831,8	1996,26	1443,36	3402,56
77	29	47	110	74	58	53	57	8,4	132,2	467,0	384,2	60,8	- 2,9	- 11,1	- 21,6	- 19,6	- 7,8	32,2	62,6	56,8	22,6	239,76	217,56	423,36
78	17	35	76	52	32	40	36	222,0	533,6	3092,5	1730,6	1142,4	- 4,9	- 23,1	- 55,6	- 41,6	- 33,8	113,2	272,4	203,8	165,6	1284,36	960,96	2312,96
79	28	62	135	95	57	70	65	15,2	15,2	11,5	1,7	77,4	- 3,9	3,9	3,4	1,4	- 8,8	- 15,2	- 13,3	- 5,5	34,3	13,26	5,46	4,76
80	11	24	51	38	19	25	26	436,8	1162,8	6498,0	3091,4	2190,2	- 20,9	- 34,1	- 80,6	- 55,6	- 46,8	712,7	1684,5	1162,0	978,1	2748,46	1895,96	4481,36
81	13	29	64	44	24	31	33	357,2	846,8	4571,1	2460,2	1747,2	- 18,9	- 29,1	- 67,6	- 49,6	- 41,8	550,0	1277,6	937,4	790,0	1967,16	1443,36	3352,96
82	16	36	77	57	29	39	38	252,8	488,4	2982,2	1339,6	1354,2	- 15,9	- 22,1	- 54,6	- 36,6	- 36,8	351,4	868,1	581,9	585,1	1206,66	808,86	1998,36
83	14	29	63	48	26	31	32	320,4	846,8	4707,3	2079,4	1584,0	- 17,9	- 29,1	- 68,6	- 45,6	- 39,8	520,9	1227,9	816,2	712,4	1996,26	1326,96	3128,16
84	22	57	126	92	51	63	63	98,0	1,0	28,2	2,0	219,0	- 9,9	- 1,1	- 5,6	- 1,6	- 14,8	10,9	55,4	15,8	146,5	6,16	1,76	8,96
85	43	65	151	109	85	74	77	123,2	47,6	376,0	237,2	367,1	11,1	6,9	19,4	15,4	19,2	76,6	215,3	170,9	213,1	133,86	106,26	298,76
86	50	102	226	157	98	116	110	327,6	1927,2	8909,5	4019,6	1036,8	18,1	43,9	94,4	63,4	32,2	794,6	1708,6	1147,5	582,8	4144,16	2783,26	5984,96
87	34	64	144	92	67	72	72	4,4	34,8	153,5	2,0	1,4	2,1	5,9	12,4	- 1,6	1,2	12,4	26,0	- 3,4	2,5	73,16	- 9,44	- 19,84
88	32	43	107	70	61	53	54	0,0	228,0	605,6	557,0	23,0	0,1	- 15,1	- 24,6	- 23,6	- 4,8	- 1,5	- 2,5	- 2,4	- 0,5	371,46	356,36	580,56
89	22	45	93	70	37	49	44	98,0	171,6	1490,7	557,0	829,4	- 9,9	- 13,1	- 38,6	- 23,6	- 28,8	129,7	382,1	233,6	285,1	505,66	309,16	910,96
90	30	43	112	80	57	57	55	3,6	228,0	384,5	185,0	77,4	- 1,9	- 15,1	- 19,6	- 13,6	- 8,8	28,7	37,2	25,8	16,7	295,96	205,36	266,56
91	64	128	281	207	122	144	137	1030,4	4886,0	22317,4	12859,6	3158,4	32,1	69,9	149,4	113,4	50,2	2243,8	4795,7	3640,1	1804,0	10443,06	7926,66	16941,96
92	26	57	131	91	56	68	63	34,8	1,0	0,4	6,8	96,0	- 5,9	- 7,1	- 0,6	- 2,6	- 9,8	6,5	3,5	15,3	57,8	0,66	2,86	- 1,26
93	29	50	113	80	59	54	59	8,4	65,6	346,3	185,0	46,2	- 2,9	- 8,1	- 18,6	- 13,6	- 6,8	23,5	53,9	39,4	19,7	150,66	110,16	252,96
94	40	70	172	126	83	83	89	65,6	141,6	1631,3	1049,8	294,5	8,1	11,9	40,4	32,4	17,2	96,4	327,2	262,4	139,3	480,76	385,56	1308,96
95	21	33	78	56	33	41	37	118,8	630,0	2874,0	1413,8	1075,8	- 10,9	- 25,1	- 53,6	- 37,6	- 32,8	273,6	584,2	409,8	357,5	1345,36	943,76	2015,36
96	22	33	88	60	42	44	44	98,0	630,0	1901,8	1129,0	566,4	- 9,9	- 25,1	- 43,6	- 33,6	- 23,8	248,5	431,6	332,6	235,6	1094,36	843,36	1464,96
87	25	39	93	73	51	45	48	47,4	364,8	1490,7	424,4	219,0	- 6,9	- 19,1	- 38,6	- 20,6	- 14,8	131,8	266,3	142,4	102,1	737,26	393,46	795,16
98	36	56	130	93	66	68	62	16,8	4,4	2,6	0,4	0,0	4,1	- 2,1	- 1,6	- 0,6	0,2	- 8,6	- 6,6	- 2,5	0,8	3,36	1,26	0,96
99	44	72	161	110	87	75	86	146,4	193,2	863,8	269,0	449,4	12,1	13,9	29,4	16,4	21,2	168,2	355,7	198,4	256,5	408,66	227,96	482,16
100	51	94	210	166	110	99	111	364,8	1288,8	6145,0	5314,2	1953,6	19,1	35,9	78,4	72,4	44,2	5685,7	1497,4	1382,8	844,2	2814,56	2599,16	5676,16
Σ	3186	5809	13161	9359	6584	6622	6539	16627,6	55112,7	267046,5	140709,1	64749,6	12,9	8,4				27152,0	62279,1	45992,7	32534,2	120422,6	85629,8	191003,04

$\overline{OC}=\overline{OD}=\overline{OB}=a$,
$\overline{AE}=b$, $\overline{AC}=m$,
$\overline{AB}=m'$, $\overline{AD}=d$.
$\overline{BC}=e$, $c=m+m'$

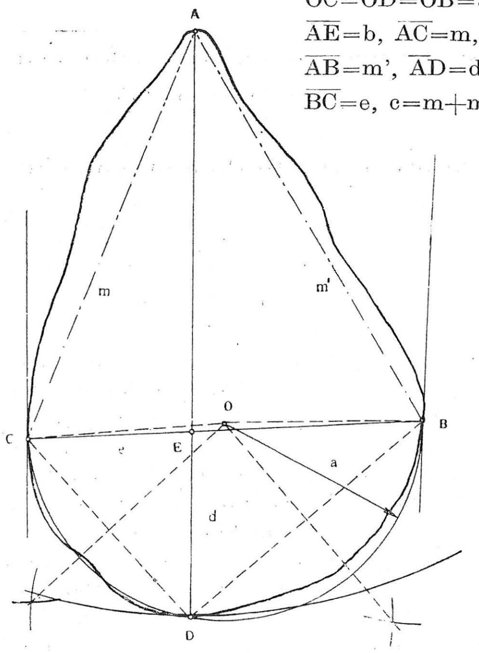

Fig: 1.

The values of the selected characters of individual members of the sample are given in table 1 along with the remaining computations which are necessary for the elaboration of mathematical relations.

The frequency distribution of individual characters is given in table 2 (a, b) for the length of the interval $h = 20$.

Table 2a

Class limits	Class mark	Class frequency of e	Cumulative frequency	Class frequency of m	Cumulative frequency	Class frequency of m	Cumulative frequency	Class frequency of b	Cumulative frequency	Class frequency of a	Cumulative frequency
1	2	3	4	3	4	3	4	3	4	3	4
11— 30	20	6	6	2	2	2	2	8	8	52	52
31— 50	40	33	39	32	34	36	38	39	47	40	92
51— 70	60	27	66	26	60	23	61	27	74	7	99
71— 90	80	21	87	25	85	23	84	16	90	1	100
91—100	100	9	96	10	95	9	93	7	97	—	100
111—130	120	3	99	2	97	4	97	3	100	—	100
131—150	140	1	100	3	100	3	100	—	100	—	100
		r=100		r=100		r=100		r=100		r=100	

Table 2b

Class limits	Class mark	Class frequency of c	Cumulative frequency
1	2	3	4
11— 30	20	—	—
31— 50	40	—	—
51— 70	60	9	9
71— 90	80	15	24
91—110	100	18	42
111—130	120	14	56
131—150	140	10	66
151—170	160	14	80
171—190	180	6	86
191—210	200	6	92
211—230	220	4	96
231—250	240	—	96
251—270	260	3	99
271—290	280	1	100
		r = 100	

Class limits	Class mark	Class frequency of d	Cumulative frequency
1	2	3	4
11— 30	20	—	—
31— 50	40	8	8
51— 70	60	24	32
71— 90	80	21	53
91—110	100	20	73
111—130	120	12	85
131—150	140	4	89
151—170	160	7	96
171—190	180	3	99
191—210	200	1	100
		r = 100	

Besides the arithmetical mean of individual elements of the sample, the modus M of individual quantities of the sample will be determined because the frequency distribution is in all cases rather asymmetrical [formulae used in this paper, unless derived, are taken from J. Janko (1947, 1948)]. Modus M will be computed according to the formula:

$$M = x_k - \frac{h}{2} \cdot \frac{n_{k+h} - n_{k-h}}{n_{k+h} - 2n_k + n_{k-h}}$$

The computed values for individual elements are given in table 3; besides the corresponding value of M, the arithmetical mean of $\bar{x}$ elements of the sample is also given for comparison:

Table 3

Element	x_k	h	n_{k+h}	n_k	n_{k-h}	M	$\bar{x}$
e	40	20	27	33	6	46,36	65,84
m	40	20	26	32	2	46,67	66,22
m′	40	20	23	36	2	44,49	65,39
d	60	20	21	24	8	66,84	93,59
b	40	20	27	39	8	44,42	58,09
a	20	20	40	52	0	26,25	31,86
c	100	20	14	18	15	98,57	131,61

Furthermore, the theoretical frequency distribution will be ascertained by the analytical method: owing to the asymmetrical distribution Poisson's exponential equation is chosen for this purpose in the following form:

$$f_x = \frac{e^{-\lambda} . \lambda^x}{x\,!} \tag{1}$$

where for x we shall successively introduce the notations of individual elements of a given sample. The length of the class interval is in all cases $h = 20$. A new variable $u_0 = 0$ is chosen for the class mark $x_i = 20$ and for each element the mean distribution λ will be ascertained from the expression:

$$\bar{x} = \frac{1}{r} \sum_{i=1}^{e} x_i\, n_i = \frac{1}{r} \sum_{i=1}^{e} (h \,.\, \omega_i + x_0)\, n_i \Rightarrow \bar{x} = h \,.\, \bar{u} + x_0 \tag{2}$$

consequently from equation (2) it follows:

$$\bar{u} = \frac{\bar{x} - x_0}{h} \tag{3}$$

x_0 being a new convenient origin.

The mean of the frequency distribution $\lambda = \bar{u}$, the corresponding notations of individual elements of the sample being introduced for $x, \bar{x}$- into expression (3).

Thus we obtain for the element e:

$$\bar{u}_e = \frac{\bar{e} - e_0}{h} = \frac{65{,}84 - 20}{20} = 2{,}292 = \lambda_e$$

similarly for the element m:

$$\bar{u}_{m}\,, = \frac{\bar{m} - m_0}{h} = 2{,}311 = \lambda_m$$

likewise, for further elements of our sample it gives:

$$\lambda_{m'} = 2{,}269\,; \quad \lambda_d = 3{,}679\,; \quad \lambda_b = 1{,}904\,; \quad \lambda_a = 0{,}593\,; \quad \lambda_c = 5{,}58$$

Now we may construct the tables of the theoretical frequency distributions, which we obtain by successive substituting into expression (1); in these tables (no. 4—8) the frequencies of individual elements according to table 2 are given in the second column n_i, the computed theoretical frequencies in the 7th column $f(x)$; therefore, it is possible to compare immediately how the ascertained frequency of each element of a given sample differs from the theoretical frequency obtained by the analytical method.

Table 4

Class mark e_i	Frequency n_i	u_i	$n_i\, u_i$	$u_i^2\, n_i$	$f(u)$	$f(e)$
20	6	0	0	0	0,1021	10,2
40	33	1	33	33	0,2327	23,3
60	27	2	54	108	0,2674	26,7
80	21	3	63	189	0,2038	20,4
100	9	4	36	144	0,1163	11,6
120	3	5	15	75	0,0552	5,5
140	1	6	6	36	0,0225	2,3
Σ	100		207	585	1,0000	100,0

As the values $\lambda_m = 2{,}311$ and $\lambda_{m'} = 2{,}269$ are very close to the value $\lambda_e = 2{,}292$ for the calculation of the theoretical frequency distribution according to formula (1), we shall not compute these values separately for the elements m and m' because the individual theoretical frequencies of the element m and m' will differ only slightly from the theoretical frequencies $f(e)$ of the element e given in table 4.

Tables of the calculation of frequencies for the remaining elements of the sample are presented further below (see tables no. 5, 6, 7, 8).

Table 5

Class mark d_i	Frequency n_i	u_i	$n_i u_i$	$n_i u_i^2$	$f(u)$	$f(d)$
20	0	0	0	0	0,0252	2,5
40	8	1	8	8	0,0938	9,4
60	24	2	48	96	0,1719	17,2
80	21	3	63	189	0,2067	20,6
100	20	4	80	160	0,1947	19,5
120	12	5	60	300	0,1428	14,3
140	4	6	24	144	0,0869	8,7
160	7	7	49	343	0,0467	4,7
180	3	8	24	192	0,0225	2,2
200	1	9	9	81	0,0088	0,9
Σ	100		455	1513	1,0000	100,0

Table 6

Class mark b_i	Frequency n_i	u_i	$n_i u_i$	$u_i^2 n_i$	$f(u)$	$f(b)$
20	8	0	0	0	0,1494	14,9
40	39	1	39	39	0,2836	28,4
60	27	2	54	108	0,2710	27,1
80	16	3	48	144	0,1714	17,1
100	7	4	28	112	0,0826	8,3
120	3	5	15	75	0,0321	3,2
140	0	6	0	0	0,0099	1,0
Σ	100		184	478	1,0000	100,0

Table 7

Class mark a_i	Frequency n_i	u_i	$n_i u_i$	$u_i^2 n_i$	$f(u)$	$f(a)$
20	52	0	0	0	0,5527	55,3
40	40	1	40	40	0,3278	32,8
60	7	2	14	56	0,0972	9,7
80	1	3	3	27	0,0192	1,9
100	0	4	0	0	0,0029	0,3
120	0	5	0	0	0,0002	0
Σ	100		57	123	1,0000	100,0

Table 8

Class mark c_i	Frequency n_i	u_i	$n_i u_i$	$n_i u_i^2$	$f(u)$	$f(c)$
20	0	0	0	0	0,0039	0,4
40	0	1	1	1	0,0213	2,1
60	9	2	18	36	0,0589	5,9
80	15	3	45	144	0,1094	10,9
100	18	4	72	288	0,1526	15,3
120	14	5	70	350	0,1703	17,0
140	10	6	60	360	0,1582	15,8
160	14	7	98	686	0,1263	12,6
180	6	8	48	384	0,0881	8,8
200	6	9	54	486	0,0546	5,5
220	4	10	40	400	0,0305	3,1
240	0	11	11	0	0,0155	1,6
260	3	12	36	432	0,0073	0,7
280	1	13	13	169	0,0031	0,3
Σ	100		566	3736	1,0000	100,0

In computing the value $f(u)$ in the column 6 of the tables 4—8 it was rounded off so that within the limits of the mentioned class marks x_i the sum of the relative frequencies $f(u)$ be equal to 1,0; the influence of this reduction on the final arrangement of the theoretical frequency distributions $f(x)$ in the column 7 of the given tables, where for x the notations of individual elements of the sample are substituted, can be neglected.

We next proceed to the determination of the correlation between the character a, which defines the radius of the circle replacing the lower border of the outline of the valve (see fig. 1), and the remaining characters.

First of all the arithmetical means of individual characters will be computed according to the formula:

$$\bar{x} = \frac{1}{r} \cdot \sum_{i=1}^{r} x_i$$

thus

$$\bar{a} = 31{,}86\,; \quad \bar{b} = 58{,}09\,; \quad \bar{c} = 131{,}61\,; \quad \bar{d} = 93{,}59\,; \quad \bar{e} = 65{,}84$$

and by means of the moment of the second order with respect to the arithmetical mean $\mu_{x,2}$ the determinative deviation σ_x; for single characters we get:

$$\mu_{a,2} = \frac{1}{r} \cdot \sum_{i=1}^{r} (a_i - a)^2 = 166{,}3 \Rightarrow \sigma_a = 12{,}87$$
$$\mu_{b,2} = 551{,}1 \Rightarrow \sigma_b = 23{,}47$$
$$\mu_{c,2} = 2670{,}46 \Rightarrow \sigma_c = 51{,}68$$
$$\mu_{d,2} = 1407{,}1 \Rightarrow \sigma_d = 37{,}51$$
$$\mu_{e,2} = 647{,}49 \Rightarrow \sigma_e = 25{,}44$$

The correlation between the radius of the curvature a and the remaining characters will be computed from the formula:

$$r_{x,y} = \frac{\Sigma (x_i - \bar{x})\,(y_i - \bar{y})}{r \cdot \sigma_x \cdot \sigma_y} \text{ [1]}$$

consequently,

$$r_{a,b} = \frac{\Sigma (a_i - \bar{a})\,(b_i - \bar{b})}{r \cdot \sigma_a \cdot \sigma_b} = \frac{27\,152{,}0}{30\,205{,}89} = 0{,}899$$

$$r_{a,c} = \frac{\Sigma (a_i - \bar{a})\,(c_i - \bar{c})}{r \cdot \sigma_a \cdot \sigma_c} = \frac{62\,279{,}1}{66\,512{,}16} = 0{,}936$$

$$r_{a\,d} = \frac{\Sigma (a_i - \bar{a})\,(d_i - \bar{d})}{r \cdot \sigma_a \cdot \sigma_d} = \frac{45\,992{,}7}{48\,275{,}37} = 0{,}953$$

$$r_{a,e} = \frac{\Sigma (a_i - \bar{a})\,(e_i - \bar{e})}{r \cdot \sigma_a \cdot \sigma_e} = \frac{32\,534{,}2}{32\,741{,}28} = 0{,}993$$

It appears that the characters b, c, d, e are in close dependence on the radius of the curvature a; therefore, we shall deal with the quadruple correlation between the character a and the remaining characters b, c, d. The character e is closely dependent of a, as can be seen from the correlation $r_{\alpha,\delta}$, therefore it can be neglected in regard to the character a in further calculations. The character e is to a great extent directly dependent on the character a, as the centres of the substituting circles with the radius a lie for the individual outlines in most cases very close to the connecting line $\overline{CB} = e$; the direct dependence of the character e on the character a may also be observed from the fact that there exists only a small deviation between e and $2a$ and that the arithmetical means of the characters e, $2a$ differ only slightly: $2\bar{a} = 63.72$, $\bar{e} = 65.84$.

First of all we deduce the necessary general relations inductively from the relations for the simple correlation and denote for these general considerations:

$$a = {}_1x\,; \quad b = {}_2x\,; \quad c = {}_3x\,; \quad d = {}_4x$$

therefore we may consider the frequency function in the case of the relative frequencies

$$f\,({}_1x\,,\ {}_2x\,,\ {}_3x\,,\ {}_4x).$$

[1]) See J. Janko, 1948, p. 101.

Each value of the variable ${}_4x$, which assumes the values ${}_4x_1$; ${}_4x_2$; ${}_4x_3$ has a certain frequency distribution of the triads of values

$${}_1x_i \; ; \quad {}_2x_j \; ; \quad {}_3x_k$$

the frequency of which will be denoted

$$n\,({}_1x_i \;,\; {}_2x_j \;,\; {}_3x_k)$$

and the relative frequency

$$f\,({}_1x_i \;,\; {}_2x_j \;,\; {}_3x_k) \;.$$

The frequency of the tetrad of values

$${}_1x_i \;,\; {}_2x_j \;,\; {}_3x_k \;,\; {}_4x_l$$

will be denoted at a certain point similarly

$$n\,({}_1x_i \;,\; {}_2x_j \;,\; {}_3x_k \;,\; {}_4x_l\,)$$

where the sequence of the elements in the parenthesis being unimportant.

Let us denote the deviations of the variable values from the arithmetical mean

$$\xi_1 \;,\; \xi_2 \;,\; \xi_3 \;,\; \xi_4$$

and the determinative deviations of the r variables

$$\sigma_1 \;,\; \sigma_2 \;,\; \sigma_3 \;,\; \sigma_4$$

then the linear regression proper set of the character ${}_1x$ in regard to the remaining characters is given by the equation [1])

$$\xi_1 = a_1 + b_{12}\xi_2 + b_{13}\xi_3 + b_{14}\xi_4 \tag{4}$$

where a_1, b_{12} , b_{13} , b_{14} are constants determined by the method of the least sum of squares so that the value ξ_1 computed from equation (4) be very close to the values which correspond to the measured values ξ_2, ξ_3, ξ_4.

If we denote the deviations

$$\xi_1 - a_1 - b_{12}\xi_2 - b_{13}\xi_3 - b_{14}\xi_4 = \xi_{1\,(2\,3\,4)} \;,$$

then the condition that the sum of squares of deviations

$$\xi^2_{1\,(2\,3\,4)}$$

be minimal, must be fulfilled.

Thus, in the case of the first derivation according to a_1; b_{12}; b_{13}; b_{14};

$$\sum_{i,j,k,l} (\xi_{1,i} - a_1 - b_{12}\xi_{2,j} - b_{13}\xi_{3,k} - b_{14}\xi_{4,l})\; n({}_1x_i;\, {}_2x_j;\, {}_3x_k;\, {}_4x_l) = 0$$

and hence it follows

$$\Sigma\xi_{1,i}\, n_{i,j,k,l} = \Sigma\xi_{2,j} n_{i,j,k,l} = \Sigma\xi_{3,k} n_{i,j,k,l} = \Sigma\xi_{4,l} n_{i,j,k,l} = 0$$

when we write briefly

$$n({}_1x_i \,,\, {}_2x_j\; {}_3x_k\; {}_4x_l) = n_{i,j,k,l}\; \sum_{i,j,k,l} = \Sigma \;;$$

consequently, we have the constant $a_1 = 0$, so that equation (4) takes the form

$$\xi_1 = b_{12}\xi_2 + b_{13}\xi_3 + b_{14}\xi_4 \tag{5}$$

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[1]) J. Janko (1948, p. 126) and J. Janko (1937, p. 76).

By further derivation we obtain

$$-2\Sigma\xi_{2,j}\,(\xi_{1,i}-b_{12}\xi_{2,j}-b_{13}\xi_{3,k}-b_{14}\xi_{4,l})\,n_{i,j,k,l}=0$$
$$-2\Sigma\xi_{3,k}\,(\xi_{1,i}-b_{12}\xi_{2,j}-b_{13}\xi_{3,k}-b_{14}\xi_{4,l})\,n_{i,j,k.l}=0$$
$$-2\Sigma\xi_{4,l}\,(\xi_{1,i}-b_{12}\xi_{2,j}-b_{13}\xi_{3,k}-b_{14}\xi_{4,l})\,n_{i,j,k,i}=0$$

thus after some conversion

$$\left.\begin{aligned}\Sigma\xi_{1,i}\xi_{2,j}n_{i,j,k,l}&=b_{12}\Sigma\xi^2_{2,j}n_{i,j,k,l}+b_{13}\Sigma\xi_{2,j}\xi_{3,k}n_{i,j,k,l}+b_{14}\Sigma\xi_{2,j}\xi_{4,l}n_{i,j,k,l}=0\\ \Sigma\xi_{1,i}\xi_{3,k}n_{i,j,k,l}&=b_{12}\Sigma\xi_{2,j}\xi_{3,k}n_{i,j,k,l}+b_{13}\Sigma\xi^2_{3,k}n_{i,j,k,l}+b_{14}\Sigma\xi_{3,k}\xi_{4,l}n_{i,j,k,l}=0\\ \Sigma\xi_{1,i}\xi_{4,l}n_{i,j,k,l}&=b_{12}\Sigma\xi_{2,j}\xi_{4,l}n_{i,j,k,l}+b_{13}\Sigma\xi_{3,k}\xi_{4,l}n_{i,j,k,l}+b_{13}\Sigma\xi^2_{4,l}n_{i,j,k,l}=0\end{aligned}\right\}\quad(6)$$

If we substitute for individual members in equation (6) the expression which works out for simple correlation[1])

$$r_{xy}=\frac{\Sigma\,\xi\,.\,\eta,}{r\,.\,\sigma_y\,.\,\sigma_x},$$

consequently in our case by the expression

$$\Sigma\xi_{1,i}\xi_{2,j}=r\,.\,r_{12}\,.\,\sigma_1\,.\,\sigma_2\,,\qquad(7)$$

we obtain after some conversion a set of equations:

$$\left.\begin{aligned}b_{12}\,.\,\sigma_2^2+b_{13}\,.\,r_{23}\,.\,\sigma_2\,.\,\sigma_3+b_{14}\,.\,r_{24}\,.\,\sigma_2\,.\,\sigma_4&=r_{12}\,.\,\sigma_1\,.\,\sigma_2\,,\\ b_{12}\,.\,r_{23}\,.\,\sigma_2\,.\,\sigma_3+b_{13}\,.\,\sigma_3^2+b_{14}\,.\,r_{34}\,.\,\sigma_3\,.\,\sigma_4&=r_{13}\,.\,\sigma_1\,.\,\sigma_3\,,\\ b_{12}\,.\,r_{24}\,.\,\sigma_2\,.\,\sigma_4+b_{13}\,.\,r_{34}\,.\,\sigma_3\,.\,\sigma_4+b_{14}\,.\,\sigma_{42}^2&=r_{14}\,.\,\sigma_1\,.\,\sigma_4\,,\end{aligned}\right\}\quad(8)$$

the solution of which gives

$$b_{12}=\frac{\begin{vmatrix}r_{12}\sigma_1\sigma_2, & r_{23}\sigma_2\sigma_3, & r_{24}\sigma_2\sigma_4\\ r_{13}\sigma_1\sigma_3, & \sigma_3^2, & r_{34}\sigma_3\sigma_4\\ r_{14}\sigma_1\sigma_4, & r_{34}\sigma_3\sigma_4, & \sigma_4^2\end{vmatrix}}{A}=\sigma_1\,.\,\sigma_2\,.\,\sigma_3^2\,.\,\sigma_4^2\,\frac{\begin{vmatrix}r_{12}, & r_{23}, & r_{24}\\ r_{13}, & 1, & r_{34}\\ r_{14}, & r_{34}, & 1\end{vmatrix}}{A}$$

$$b_{13}=\frac{\begin{vmatrix}\sigma_2^2\,, & r_{12}\sigma_1\sigma_2, & r_{24}\sigma_2\sigma_4\\ r_{23}\sigma_2\sigma_3, & r_{13}\sigma_1\sigma_3, & r_{34}\sigma_3\sigma_4\\ r_{24}\sigma_2\sigma_4, & r_{14}\sigma_1\sigma_4, & \sigma_4^2\end{vmatrix}}{A}=\sigma_1\,.\,\sigma_2^2\,.\,\sigma_3\,.\,\sigma_4^2\,\frac{\begin{vmatrix}1, & r_{12}, & r_{24}\\ r_{23}, & r_{13}, & r_{34}\\ r_{24}, & r_{14}, & 1\end{vmatrix}}{A}$$

$$b_{14}=\frac{\begin{vmatrix}\sigma_2^2, & r_{23}\sigma_2\sigma_3, & r_{12}\sigma_1\sigma_2\\ r_{23}\sigma_2\sigma_3, & \sigma_3^2, & r_{13}\sigma_1\sigma_3\\ r_{24}\sigma_2\sigma_4, & r_{34}\sigma_3\sigma_4, & r_{14}\sigma_1\sigma_4\end{vmatrix}}{A}=\sigma_1\,.\,\sigma_2^2\,.\,\sigma_3^2\,.\,\sigma_4\,\frac{\begin{vmatrix}1, & r_{23}, & r_{12}\\ r_{23}, & 1 & r_{13}\\ r_{24}, & r_{34}, & r_{14}\end{vmatrix}}{A}$$

$$A=\begin{vmatrix}\sigma_2^2, & r_{23}\sigma_2\sigma_3, & r_{24}\sigma_2\sigma_4\\ r_{23}\sigma_2\sigma_3, & \sigma_3^2, & r_{34}\sigma_3\sigma_4\\ r_{24}\sigma_2\sigma_4, & r_{34}\sigma_3\sigma_4, & \sigma_4^2\end{vmatrix}=\sigma_2^2\sigma_3^3\sigma_4^2\begin{vmatrix}1, & r_{23}, & r_{24}\\ r_{23}, & 1, & r_{34}\\ r_{24}, & r_{34}, & 1\end{vmatrix}$$

[1]) See J. Janko, 1948, p. 101.

thus finally,

$$b_{12} = \frac{\sigma_1}{\sigma_2} \cdot \frac{\begin{vmatrix} r_{12}, r_{23}, r_{24} \\ r_{13}, 1, r_{34} \\ r_{14}, r_{34}, 1 \end{vmatrix}}{\begin{vmatrix} 1, r_{23}, r_{24} \\ r_{23}, 1, r_{34} \\ r_{24}, r_{34}, 1 \end{vmatrix}}, \quad b_{13} = \frac{\sigma_1}{\sigma_3} \cdot \frac{\begin{vmatrix} 1, r_{12}, r_{24} \\ r_{23}, r_{13}, r_{34} \\ r_{24}, r_{14}, 1 \end{vmatrix}}{\begin{vmatrix} 1, r_{23}, r_{24} \\ r_{23}, 1, r_{34} \\ r_{24}, r_{34}, 1 \end{vmatrix}}, \quad b_{14} = \frac{\sigma_1}{\sigma_4} \cdot \frac{\begin{vmatrix} 1, r_{23}, r_{12} \\ r_{23}, 1, r_{13} \\ r_{24}, r_{34}, r_{14} \end{vmatrix}}{\begin{vmatrix} 1, r_{23}, r_{24} \\ r_{23}, 1, r_{34} \\ r_{24}, r_{34}, 1 \end{vmatrix}}$$

In order to find the formula for our quadruple correlation we shall start with the corresponding formula for the mean square deviation

$$s_{xy}^2 = \sigma_y^2 \cdot (1 - r_{yx}^2)$$

which can be in our notation written in the form

$$s_{12}^2 = \sigma_1^2 \cdot (1 - r_{12}^2) \tag{10}$$

because the sequence of the indexes is not important.

Similarly, as in the case of equation (5), we obtain in our case for the mean square deviation the expression

$$s_{1(234)}^2 = \frac{1}{r} \Sigma (\xi_1 - b_{12}\xi_2 - b_{13}\xi_3 - b_{14}\xi_4)^2 n_{i,j,k,l}; \tag{11}$$

if we raise to the second power, express again the value of the variable from the arithmetical mean through equation (7) and introduce the corresponding values from equations (8), we obtain successively

$$s_{1(234)}^2 = \frac{1}{r} \Sigma [(\xi_1 - b_{12}\xi_2)^2 - 2(\xi_1 - b_{12}\xi_2)(b_{13}\xi_3 + b_{14}\xi_4) + (b_{13}\xi_3 + b_{14})^2]\, n_{i,j,k,l} =$$

$$= \sigma_1^2 - 2b_{12}(b_{12}\sigma_2^2 + b_{13}r_{23}\sigma_2\sigma_3 + b_{14}r_{24}\sigma_2\sigma_4) + b_{12}^2\sigma_2^2 - 2b_{13}(b_{12}r_{23}\sigma_2\sigma_3 + b_{13}\sigma_3^2 + b_{14}r_{34}\sigma_3\sigma_4) -$$
$$- 2b_{14}(b_{12}r_{24}\sigma_2\sigma_4 + b_{13}r_{34}\sigma_3\sigma_4 + b_{14}\sigma_4^2) + 2b_{12}b_{13}r_{23}\sigma_2\sigma_3 + 2b_{12}b_{14}r_{24}\sigma_2\sigma_4 + b_{13}^2\sigma_3^2 +$$
$$+ 2b_{13}b_{14}r_{34}\sigma_3\sigma_4 + b_{14}^2\sigma_4^2 = \sigma_1^2 - b_{12}^2\sigma_2^2 - b_{13}^2\sigma_3^2 - b_{14}^2\sigma_4^2 - 2b_{12}b_{13}r_{23}\sigma_2\sigma_3 -$$
$$- 2b_{12}b_{14}r_{24}\sigma_3\sigma_4 - 2b_{13}b_{14}r_{34}\sigma_3\sigma_4$$

and writing instead of b_{12}, b_{13}, b_{14} their values according to equation (9) in the form

$$b_{12} = \frac{\sigma_1}{\sigma_2} \cdot \frac{B_{12}}{A}, \quad b_{13} = \frac{\sigma_1}{\sigma_3} \cdot \frac{B_{13}}{A}, \quad b_{14} = \frac{\sigma_1}{\sigma_4} \cdot \frac{B_{14}}{A}$$

it gives after putting before parentheses and conversion

$$s_{1(234)}^2 = \sigma_1^2 - \frac{\sigma_1^2}{A^2} \cdot \left[B_{12}^2 + B_{13}^2 + B_{14}^2 + 2B_{12}B_{13}r_{23} + 2B_{12}B_{14}r_{24} + 2B_{13}B_{14}r_{34} \right],$$

thus finally

$$s_{1(234)}^2 = \sigma_1^2 \left[1 - \frac{B_{12}^2 + B_{13}^2 + B_{14}^2 + 2B_{12}B_{13}r_{23} + 2B_{12}B_{14}r_{24} + 2B_{13}B_{14}r_{34}}{A^2} \right];$$

if we compare this equation with equation (10) we get the coefficient of the quadruple correlation in the form

$$r_{1(234)} = \left[\frac{B_{12}^2 + B_{13}^2 + B_{14}^2 + 2B_{12}B_{13}r_{23} + 2B_{12}B_{14}r_{24} + 2B_{13}B_{14}r_{34}}{A^2} \right]^{1/2}$$

and writing instead of the individual indexes the original notations of the characters of our sample we have:

$$r_{a(bcd)} = \left[\frac{B_{ab}^2 + B_{ac}^2 + B_{ad}^2 + 2B_{ab}B_{ac}r_{bc} + 2B_{ab}B_{ad}r_{bd} + 2B_{ac}B_{ad}r_{cd}}{A_a^2}\right]^{1/2} \qquad (12)$$

In order to be able to use formula (12) for computing the quadruple correlation between the character a and the characters b, c, d of our sample, it is necessary to find the simple correlations of the remaining combinations of couples of characters: r_{bc}, r_{bd}, r_{cd}.

We get successively the following values:

$$r_{bc} = \frac{\Sigma(b_i - \bar{b})(c_i - \bar{c})}{r \cdot \sigma_b \cdot \sigma_c} = \frac{120\,422{,}60}{121\,292{,}96} = 0{,}993$$

$$r_{bd} = \frac{85\,629{,}80}{88\,035{,}97} = 0{,}973, \quad r_{cd} = \frac{191\,003{,}04}{193\,851{,}68} = 0{,}985$$

Now it will be computed successively:

$$B_{ab}^2 = \begin{vmatrix} r_{ab}, & r_{bc}, & r_{bd} \\ r_{ac}, & 1, & r_{cd} \\ r_{ad}, & r_{cd}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 0.899, & 0{,}993, & 0{,}973 \\ 0{,}936, & 1, & 0{,}985 \\ 0{,}953, & 0{,}985, & 1 \end{vmatrix}^2 = (-0{,}000748)^2 = 0{,}000000559504$$

$$B_{ac}^2 = \begin{vmatrix} 1, & r_{ab}, & r_{bd} \\ r_{bc}, & r_{ac}, & r_{cd} \\ r_{bd}, & r_{ad}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 1, & 0{,}889, & 0{,}973 \\ 0{,}993, & 0{,}936, & 0{,}985 \\ 0{,}973, & 0{,}953, & 1 \end{vmatrix}^2 = (0{,}0008339)^2 = 0{,}0000006953892$$

$$B_{ad}^2 = \begin{vmatrix} 1, & r_{bc}, & r_{ab} \\ r_{bc}, & 1, & r_{ac} \\ r_{bd}, & r_{cd}, & r_{ad} \end{vmatrix}^2 = \begin{vmatrix} 1, & 0{,}993, & 0{,}899 \\ 0{,}993, & 1, & 0{,}936 \\ 0{,}973, & 0{,}985, & 0{,}953 \end{vmatrix}^2 = (0{,}000281)^2 = 0{,}000000078961$$

$2 \cdot B_{ab} \cdot B_{ac} \cdot r_{bc} = 2 \cdot (-0{,}000748) \cdot 0{,}0008339 \cdot 0{,}993 = -0{,}000001238782,$
$2 \cdot B_{ab} \cdot B_{ad} \cdot r_{bd} = 2 \cdot (-0{,}000748) \cdot 0{,}000281 \cdot 0{,}973 = -0{,}0000004090258,$
$2 \cdot B_{ac} \cdot B_{ad} \cdot r_{cd} = 2 \cdot 0{,}0008339 \cdot 0{,}000281 \cdot 0{,}985 = 0{,}000000461622,$

$$A_a^2 = \begin{vmatrix} 1, & r_{bc}, & r_{bd} \\ r_{bc}, & 1, & r_{cd} \\ r_{bd}, & r_{cd}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 1, & 0{,}993, & 0{,}973 \\ 0{,}993, & 1, & 0{,}985 \\ 0{,}973, & 0{,}985, & 1 \end{vmatrix}^2 = 0{,}0003894^2 = 0{,}000000151632$$

Substitution in formula (12) gives:

$$r_{a(bcd)} = \sqrt{\frac{147\,668}{151\,632}} = \sqrt{0{,}973858} = 0{,}987$$

By cyclic replacement of the characters of our sample further quadruple correlation will be computed on the basis of equation (12): $r_{b(bcd)}$, $r_{b(cad)}$, $r_{d(abc)}$

According to formula (12) we have:

$$r_{d(abc)} = \left[\frac{B_{da}^2 + B_{db}^2 + B_{dc}^2 + 2 \cdot B_{da} \cdot B_{db} \cdot r_{ab} + 2 \cdot B_{da} \cdot B_{dc} \cdot r_{ac} + 2 \cdot B_{db}B_{dc}r_{bc}}{A_d^2}\right]^{1/2}, \qquad (13)$$

where

$$A_d^2 = \begin{vmatrix} 1, r_{ab}, r_{ac} \\ r_{bd}, 1, r_{bc} \\ r_{ac}, r_{bc}, 1 \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}899, 0{,}936 \\ 0{,}899, 1, 0{,}993 \\ 0{,}936, 0{,}993, 1 \end{vmatrix}^2 = (0{,}000802)^2 = 0{,}000000643204$$

$$B_{da}^2 = \begin{vmatrix} r_{da}, r_{ab}, r_{ac} \\ r_{db}, 1, r_{bc} \\ r_{dc}, r_{bc}, 1 \end{vmatrix}^2 = \begin{vmatrix} 0{,}953, 0{,}899, 0{,}936 \\ 0{,}973, 1, 0{,}993 \\ 0{,}985, 0{,}993 & 1 \end{vmatrix}^2 = (0{,}000281)^2 = 0{,}000000078916$$

$$B_{db}^2 = \begin{vmatrix} 1, r_{da}, r_{ac} \\ r_{ab}, r_{db}, r_{bc} \\ r_{ac}, r_{dc}, 1 \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}953, 0{,}936 \\ 0{,}899, 0{,}973, 0{,}993 \\ 0{,}936, 0{,}985, 1 \end{vmatrix}^2 = (0{,}000313)^2 = 0{,}000000097969$$

$$B_{dc}^2 = \begin{vmatrix} 1, r_{ab}, r_{da} \\ r_{ab}, 1, r_{db} \\ r_{ac}, r_{bc}, r_{dc} \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}899, 0{,}953 \\ 0{,}899, 1, 0{,}973 \\ 0{,}936, 0{,}993, 0{,}985 \end{vmatrix}^2 = (0{,}0002193)^2 = 0{,}00000004809249$$

$2 . B_{da} . B_{db} . r_{ab} = 2 . 0{,}000281 . 0{,}000313 . 0{,}899 = 0{,}000000158139$
$2 . B_{da} . B_{dc} . r_{ac} = 2 . 0{,}000281 . 0{,}0002193 . 0{,}936 = 0{,}0000001153588$
$2 . B_{db} . B_{dc} . r_{bc} = 2 . 0{,}000313 . 0{,}0002193 . 0{,}993 = 0{,}0000001363208$

substitution in formula (13) gives:

$$r_{d(abc)} = \sqrt{\frac{634\ 841}{643\ 204}} = \sqrt{0{,}986\ 998} = 0{,}993$$

According to equation (12) we obtain futher

$$r_{b(cda)} = \left[\frac{B_{bc}^2 + B_{bd}^2 + B_{ba}^2 + 2 . B_{bc} . B_{bd} . r_{cd} + 2 . B_{bc} . B_{ba} . r_{ac} + 2 . B_{bd} . B_{ba} . r_{ad}}{A_b^2}\right]^{1/2}, \tag{14}$$

where

$$A_b^2 = \begin{vmatrix} 1, r_{cd}, r_{ac} \\ r_{cd}, 1, r_{ad} \\ r_{ac}, r_{ad}, 1 \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}985, 0{,}936 \\ 0{,}985, 1, 0{,}953 \\ 0{,}936, 0{,}953, 1 \end{vmatrix}^2 = (0{,}002726)^2 = 0{,}000007431076$$

$$B_{bc}^2 = \begin{vmatrix} r_{bc}, r_{cd}, r_{ac} \\ r_{bd}, 1, r_{ad} \\ r_{ab}, r_{ad}, 1 \end{vmatrix}^2 = \begin{vmatrix} 0{,}993, 0{,}985, 0{,}936 \\ 0{,}973, 1, 0{,}953 \\ 0{,}899, 0{,}953, 1 \end{vmatrix}^2 = (0{,}0030991)^2 = 0{,}000009604421$$

$$B_{bd}^2 = \begin{vmatrix} 1, r_{bc}, r_{ac} \\ r_{cd}, r_{bd}, r_{ad} \\ r_{ac}, r_{ab}, 1 \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}993, 0{,}936 \\ 0{,}985, 0{,}973, 0{,}953 \\ 0{,}936, 0{,}899, 1 \end{vmatrix}^2 = (0{,}000313)^2 = 0{,}000000097969$$

$$B_{ba}^2 = \begin{vmatrix} 1, r_{cd}, r_{bc} \\ r_{cd}, 1, r_{bd} \\ r_{ac}, r_{ad}, r_{ab} \end{vmatrix}^2 = \begin{vmatrix} 1, 0{,}985, 0{,}993 \\ 0{,}985, 1, 0{,}973 \\ 0{,}936, 0{,}953, 0{,}899 \end{vmatrix}^2 = (-0{,}000748)^2 = 0{,}000000559504$$

$2B_{bc} . B_{bd} . r_{cd} = 2 . 0{,}0030991 . 0{,}000313 . 0{,}985 = 0{,}000001910936$
$2B_{bc} . B_{ba} . r_{ac} = 2 . 0{,}0030991 . (-0{,}000748) . 0{,}936 = -0{,}000004339539$
$2B_{bd} . B_{ba} . r_{ad} = 2 . 0{,}000313 . (-0{,}000748) . 0{,}953 = -0{,}000000446240$

consequently,

$$r_{b(cda)} = \sqrt{\frac{7387051}{7431076}} = \sqrt{0,994075} = 0,997$$

In the last case we have according to equation (12)

$$r_{c(dab)} = \left[\frac{B_{cd}^2 + B_{ca}^2 + B_{cb}^2 + 2\,.\,B_{cd}\,.\,B_{ca}\,.\,r_{ad} + 2\,.\,B_{cd}\,.\,B_{bc}\,.\,r_{bd} + 2\,.\,B_{ca}\,.\,B_{cb}\,.\,r_{ab}}{A_b^2} \right]^{1/2}, \quad (15)$$

in which

$$A_c^2 = \begin{vmatrix} 1\,, & r_{ad}, & r_{bd} \\ r_{ad}, & 1\,, & r_{ab} \\ r_{bd}, & r_{ab}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 & , 0,953, & 0,973 \\ 0,953, & 1 & , 0,899 \\ 0,973, & 0,899, & 1 \end{vmatrix}^2 = (0,0040906)^2 = 0,000016733,$$

$$B_{cd}^2 = \begin{vmatrix} r_{cd}, & r_{ad}, & r_{bd} \\ r_{ac}, & 1\,, & r_{ab} \\ r_{bc}, & r_{ab}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 0,985, & 0,953, & 0,973 \\ 0,936, & 1 & , 0,899 \\ 0,993, & 0,899, & 1 \end{vmatrix}^2 = (0,0002193)^2 = 0,00000004809249,$$

$$B_{ca}^2 = \begin{vmatrix} 1\,, & r_{cd}, & r_{bd} \\ r_{ad}, & r_{ac}, & r_{ab} \\ r_{bd}, & r_{bc}, & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 & , 0,985, & 0,973 \\ 0,953, & 0,936, & 0,899 \\ 0,973, & 0,993, & 1 \end{vmatrix}^2 = (0,0008339)^2 = 0,0000006953892,$$

$$B_{cb}^2 = \begin{vmatrix} 1\,, & r_{ad}, & r_{cd} \\ r_{ad}, & 1\,, & r_{ac} \\ r_{bd}, & r_{ab}, & r_{bc} \end{vmatrix}^2 = \begin{vmatrix} 1 & , 0,953, & 0,985 \\ 0,953, & 1 & , 0,936 \\ 0,973, & 0,899, & 0,993 \end{vmatrix}^2 = (0,0030991)^2 = 0,000009604421,$$

$$2\,.\,B_{cd}\,.\,B_{ca}\,.\,r_{ad} = 2\,.\,0,0002193\,.\,0,0008339\,.\,0,953 = 0,0000003485583,$$
$$2\,.\,B_{cd}\,.\,B_{cb}\,.\,r_{bd} = 2\,.\,0,0002193\,.\,0,0030991\,.\,0,973 = 0,000001322565,$$
$$2\,.\,B_{ca}\,.\,B_{cb}\,.\,r_{ab} = 2\,.\,0,0008339\,.\,0,0030991\,.\,0,899 = 0,000004646642$$

and after the substitution into equation (15) we have

$$r_{c(dab)} = \sqrt{\frac{166\,657}{167\,330}} = \sqrt{0,995978} = 0,998$$

The single values of the correlation coefficients may be now arranged as follows:

$$r_{ab} = 0,899 \qquad r_{a(bcd)} = 0,987$$
$$r_{ac} = 0,936 \qquad r_{b(cda)} = 0,997$$
$$r_{ad} = 0,953 \qquad r_{c(abd)} = 0,998$$
$$r_{bc} = 0,993 \qquad r_{d(abc)} = 0,993$$
$$r_{bd} = 0,973$$
$$r_{cd} = 0,985$$

Thus, as was expected, the multiple correlation appears much closer than the simple correlation; this also proves that the individual characters a, b, c, d, of our sample were selected advantageously and their close relation suggests the homogeneity of the chosen sample.

Conclusion

From what was stated above it becomes evident that the studied collection of forty-three Barrande's Lower Paleozoic species coming from different stratigraphical levels and designated by cumulative name *Mytilus* (non *Mytilus* Linné, 1758) cannot be differenciated on the basis of the outlines by means of the used biometrical methods. Even less is it possible to distinguish these forms on the basis of purely subjective visual observations of the external habitus.

Therefore it can be stated that the above mentioned mytiloid forms are from the taxonomical point of view undefinable and from the nomenclatorical point of view indistinguishable. Consequently, it seems evident that these forms cannot be placed with certainty to any already fixed genera and species, if those are characterized also by the structure of their hinge apparatus and represent natural taxa. This is verified also by the finding of a complete mytiloid specimen, in which we managed to reveal the hinge apparatus of a special character, which even justifies the establishing of a new genus (see Růžička — Prantl, 1961), although this form does not differ in its external habitus from the remaining forms designated by J. Barrande by cumulative name *Mytilus*.

Thus it may ve stated that the forms which can be distinguished only on the basis of subjective visual observations according to individual differences in the outline and surface sculpture are only artificial groups (*parataxa*) without systematical significance. Consequently, they cannot be used in solving evolutionary or phylogenetic problems. The less can they be used as biostratigraphical criteria.

Till further notice, unless we succeed in distinguishing these forms by means of perfected biometrical methods, paleontology must put up with a mere statement that in these cases it deals with undefinable mytiloid forms. This fact will then be expressed by the use of open nomeclature in the most general sense (*mytiloid form inc. sed.*)

There is no doubt that this case of generic and specific undefinability of mytiloid forms is not an isolated case among Paleozoic similarly preserved pelecypods and that it occurs also in some other groups in which the hinge apparatus in not observable owing to the unfavourable state of preservation.

At present we cannot determine to what extent the forming of the external habitus of all mytiloid pelecypods took place according to the ascertained principles, however, we believe that further comparative studies will enable us to solve this basic problem.

Nevertheless, the result of our study proves that the mytiloid type of pelecypods (characterized by its conspicuous outline) forms from the general evolutionary point of view morphologically very stabilized group of a great chronostratigraphical range (Silurian to Recent). This group may include homeomorphically related forms of different phylogenetic origin, this evolution being conditioned by homologic ecological factors.

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*) *Mytilus adjunctus, M. adornatus, M. amygdala, M.? appendens, M. budnianus, M. buridani, M. capillosus, M. carens, M. confraternus, M. consobrinus, M. consors, M. conspicuus, M. cuneus, M. discretus, M. ? elaborans, M. elongans, M. esuriens, M. excisus, M. faustulus, M. humilis, M. insectus, M.? insolitus, M.? insons, M.? laceratus, M. longior, M. nasutus, M. parens, M. patiens, M.? praecox, M. protendens, M. pyrum M. radius, M. raptus, M. rarus, M. rostratus, M. sagittalis, M. scapha, M. scarabeus, M. sector, M. securis, M. spatula, M. suavis, M. volitans.*

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